Correspondence
of
N. Bernoulli with W. 's Gravesande
Regarding Divine Providence*

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1712

Addition of the editor: We have seen in the note that I have placed at the beginning of the preceding Dissertation, that it had been composed on the occasion of the proof in favor of Providence, that Dr. *Arbuthnot* had drawn from the regularity that one observes in the number of boys & of girls who are born each year. I have remarked that one would not agree generally on the validity of the proof. Mr. Nicolas Bernoulli who, on a trip that he made to Holland, conversed with Mr. 's Gravesande on it, was of the number of those who were not convinced at all of the force of this argument. Here is that which he has written on it from London in a letter dated 30 7^{bre} 1712.

N. Bernoulli to W. 's Gravesande

At London 30 September 1712

I have disputed quite at length with Messrs. *Craig* & *Burnet*, on this argument for divine Providence drawn from the regularity which one observes in the numbers of males & of females, who are born each year in London, & of which you have spoken to me, when I have had the honor to greet you at la Hague. I have finally demonstrated to them that in a great number of tokens with two faces (heads & tails) tossed into the air, there is a quite great probability that nearly the half will be heads, & the half tails, & that consequently it must not pass for a miracle, that one sees that there is born each year nearly an equal number of males & of females, & that to the contrary this must have been a miracle if this did not happen. The fault of Mr. *Arbuthnot* consists in this that he has taken this equality too precise, & that he has not at all observed that in his list of 82 years the limits are so great, that he has a very great probability that the number of males & of females will fall within these limits. I have found by making the calculation that it is more than 7000 times more probable that this number will fall between these limits than outside. Mr. Burnet said to me that Mr. *Nieuwentyt* must

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print this supposed argument in a Book which he is going to give to the public, this is why I believe that Mr. *Nieuwentyt* will be very glad to be forewarned to the invalidity of this argument.

Addition of the editor: Mr. *'s Gravesande* takes the defense of Dr. *Arbuthot*, in a response that he made to Mr. *Bernoulli*, of which I have not found the copy at all among his manuscripts, but of which one will be able to conjecture the content, by the reply of Mr. *Bernoulli*, in a letter written from The Hague dated 9 November 1712, & which is here.

N. Bernoulli to W. 's Gravesande

At The Hague 9 November 1712

I would have well wished that our sentiments on the supposed argument of divine Providence had been the same. You said in a Letter that you have given me the honor to send to me in London, that it is the same thing, that if one wished to find the lot of the one who would wager that a token will fall 82 consecutive times on the same face. I pray you, Sir, to reflect again a little on this; I am persuaded that you will find easily that you have deceived yourself. You have only to consider these two points. 1°. That it is not one token, that one casts 82 times into the air, but that there are many, for example 1000. 2°. That among these 1000 tokens, cast 82 times into the air, there are always *nearly* the half, not precisely, which fall on one face, & the other half onto the other, & that nonetheless it is not necessary that the same token which has fallen the first time for example *heads*, falls also the second time *heads*, because it is able to fall *tails*, & another in exchange, which had fallen *tails* before, is able to fall this time *heads*, & thus they are able to vary in many ways. I send you a copy of the demonstration which I have given to Mr. *Burnet*; I hope that it will clarify to you & will convince you entirely. &c.

Addition of the editor: I join here this demonstration of which Mr. *Bernoulli* speaks. The author has inserted it in the second edition of l'*Essai d'Analyse sur les Jeux de hazard*, by Mr. *de Montmort*, page 373 & 378. But it appeared under a slightly different form, thus one will see with pleasure such as it was first composed in the heat of the dispute. It merits much to be read; the Author proves very well with that sagacity which was proper to him, that there is a very great probability that the number of males & of females will fall between some limits yet smaller than those that one has observed at London, during 82 years.

Excerpt from the Letter of N. BERNOULLI *to* D.W. BURNET R. S. S.

In order that I may fulfill the promises by my demonstration of them, which I have said in person against the argument for Divine Providence selected from the regularity observed in the birth of each sex. As soon as I crossed over The Hague Mr. 'sGravesande first had spoken to me concerning this argument, affirming vigorously to be impossible, that with a great number of tokens having two faces white & black cast, as many white faces must fall as black, & more improbable by far besides, that the same with many trials must happen in order. But I had retorted immediately, indeed vigorously it to be improbable, the number of faces will be *precisely* equal, but to appear to me, a sound mind ought declare to each what is the greatest probability, that it may approach a ratio of near equality. I added my father's brother *Jac. Bernoulli* in his posthumous Treatise on the Art of Conjecturing, which now is being printed in Basel, to have demonstrated generally, but if we wish to investigate the number of cases by which any event at all happens or not happens through trials, thus we must be able to increase the number of observations, so that in the end that which is given by probability is the more probable, to have revealed to us the true number of cases; and even to be certain to me, because with the assumed an equal facility to be in any one birth, so that male or female may be born, by which the greater number of births will have been, so much the more rather the number of males & females must be approaching near a ratio of equality. When You afterwards should have shown to me the same argument inserted into the Transactions, I have seen, & You likewise have seen, the Author of this argument likewise to have made use of a method toward establishing that argument of yours, which I toward overturning. The former has observed rightly the numbers of cases in which nearly an equal number of males & females is born, to be expressed by the terms on both sides neighboring the middle term of this series $M^n + \frac{n}{1} \times M^{n-1}F + \frac{n}{1} \times \frac{n-1}{2} \times M^{n-2}F^2 + \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times M^{n-3}F^3$+&c. But in this only he has erred, because just as he has assumed certain & unquestionable, the number of males & females always to be distant too little from exact equality, and the limits precisely to be very small, in order that the sum of the terms to be obtained from either side of the middle may have an exceedingly small ratio to the sum of the remaining terms; if he should have satisfied himself to examine his list of 82 years, by all means he should have discovered the contrary, certainly the limits to be so great, that the magnitude is the probability of the numbers of males & females going to fall between much smaller limits besides. In regard to the demonstration of such matter I shall send ahead the following.

LEMMA

If in that series $M^n + \frac{n}{1} \times M^{n-1}F + \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times M^{n-2}F^2$&c. where the exponent n is equal to the binomial $M + F$ itself, two terms may be obtained, of which one is in order $F + 1$, & the other preceding this by the interval L of terms or the term in order $F - L + 1$, and the former has to that the ratio as m to 1; I say the sum of all terms included by these two terms together with the term $F - L + 1$ itself, to the sum of all remaining preceding will have a greater ratio than $m - 1$ to 1.

DEMONSTRATION

From the law of progression the term $F + 1$ is

$$\frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \cdots \frac{n-F+1}{F} \times M^{n-F}F^F,$$

& the term $F - L + 1$,

$$\frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \cdots \frac{n-F+L+1}{F-L} \times M^{n-F+L}F^{F-L},$$

hence created by division the ratio of the former to the latter is discovered as

$$\frac{n-F+L}{F-L+1} \times \frac{n-F+L-1}{F-L+2} \times \frac{n-F+L-2}{F-L+3} \times \cdots \frac{n-F+1}{F} \times \frac{F}{M}\Bigg]^{L}$$

to 1, or by putting M for $n-F$, as

$$\frac{M+L}{F-L+1} \times \frac{M+L-1}{F-L+2} \times \frac{M+L-2}{F-L+3} \times \cdots \frac{M+1}{F} \times \frac{F}{M}\Bigg]^{L} \text{ to 1.}$$

In like manner the ratio of the term F to the term $F-L$ will be

$$\frac{M+L+1}{F-L} \times \frac{M+L}{F-L+1} \times \frac{M+L-1}{F-L+2} \times \cdots \frac{M+2}{F-1} \times \frac{F}{M}\Bigg]^{L} \text{ to 1;}$$

but this ratio is greater by the preceding on account of the several factors of this greater than the several factors of that. Thus also the ratio of the term $F-1$ to the term $F-L-1$ will have a greater ratio than the term F to the term $F-L$; and thus in turn by taking in reverse the ratio of any term whatever to any other preceding term whatever always will be less than the former, which is between the terms immediately preceding. Whence if all terms of that series with the term $F+1$ excepted are distinguished into Classes of which any whatever may contain L terms, by starting to enumerate from the term F, the first term of the first Class will have to the first term of the second Class a greater ratio than the term $F+1$ to the term $F-L+1$; & the second term of the first Class to the second of the second a greater ratio still, and the third to the third greater still & thus hereafter; consequently all terms simultaneously of the first Class to all simultaneously obtained of the second will have in addition a greater ratio than the term $F+1$ has to the term $F-L+1$. In the same manner it is demonstrated all terms of the second Class to all terms of the third, likewise all of the third to all of the fourth, all of the fourth to all of the fifth &c. to have a greater ratio than the term $F+1$ to the term $F-L+1$. Hence if this ratio of the term $F+1$ to the term $F-L+1$ be as m to 1, & the sum of the terms of the first Class may be called S, the sum of the terms of the second will be less than $\frac{S}{m}$, of the third less than $\frac{S}{mm}$, of the fourth less than $\frac{S}{m^3}$, &c. and indeed the sum of all the Classes with the first excepted, even if especially the number of Classes should be infinite, will be less than this series $\frac{S}{m} + \frac{S}{mm} + \frac{S}{m^3} + \frac{S}{m^4}$, &c. continued to infinity this is less than $\frac{S}{m-1}$, whence finally it follows the first Class to the sum of the remaining altogether to have a greater ratio than $m-1$ to 1. Q.E.D.

Now the principal problem itself follows, which generally is able to be proposed thus: With specified that in each birth are M cases against F, that a male may be born rather than a female, to discover how much must be the probability, that among $M+F$ infants the number of males may not differ from the number M to a greater extent than by some given number L; that is to discover what kind be the probability of the number of males being no greater than $M+L$, nor less than $M-L$, and of the number of females not less than $F-L$, nor greater than $F+L$. It is clear from the Doctrine of Combinations, all cases in which it is able to happen, that the number of males is greater than M, but less than $M+L+1$, and the number of women less than F, but greater than $F-L-1$, to be expressed through all terms of the first Class of the series of the preceding Lemma; & the case that the number of males be greater than $M+L$, and females less than $F-L$, through the sum of all the preceding terms; but it is demonstrated in the said Lemma, that if the ratio of the term $F+1$ in order to the term $F-L+1$ may be called m, the sum of all terms of the first Class comprehended between the terms $F+1$ & $F-L$, to the sum of all preceding terms has a ratio greater than $m-1$ to 1. Nothing other therefore remains to be discovered than the value of m itself, or of this series

$$\frac{M+L}{F-L+1}\times\frac{M+L-1}{F-L+2}\times\frac{M+L-2}{F-L+3}\times\cdots\frac{M+1}{F}\times\frac{F}{M}\Big\rceil^{L},$$

which as in the demonstration of the Lemma we have seen expresses the ratio of the term $F+1$ to the term $F-L+1$. Now since the operation for discovering that value exactly must be exceedingly lengthy, I shall give the method by which however it will be permitted to approximate the true value nearly enough. The individual factors of this series

$$\frac{M+L}{F-L+1}\times\frac{M+L-1}{F-L+2}\times\frac{M+L-2}{F-L+3}\times\cdots\frac{M+1}{F}$$

may be supposed to be in Geometric progression, and of themselves the Logarithms in Arithmetic progression, that supposition deviates very little from the truth, especially when M & F are great numbers; therefore the sum of all these Logarithms will be

$$\frac{1}{2}L\times\overline{\log\frac{M+L}{F+L+1}+\log\frac{M+1}{F}},$$

it is, the sum of the Logarithms of the first & last factor multiplied by half the number of terms, to which if the Logarithm of $\frac{F}{M}\big\rceil^{L}$ itself is added or $L\times\log\frac{F}{M}$, there will be had

$$\frac{1}{2}L\times\overline{\log\frac{M+L}{F+L+1}+\log\frac{M+1}{F}}+L\times\log\frac{F}{M},$$

or

$$\frac{1}{2}L\times\overline{\log\frac{M+L}{F+L+1}+\log\frac{M+1}{F}+\log\frac{F}{M}}$$

for the Logarithm of the quantity m, and exactly

$$\overline{\frac{M+L}{F+L+1}\times\frac{M+1}{F}\times\frac{F}{M}}\Big\rceil^{\frac{1}{2}L},$$

hence the probability that the number of males be greater than M but less than $M+L+1$ to the probability that it is greater than $M+L$ has greater ratio than

$$\overline{\frac{M+L}{F+L+1}\times\frac{M+1}{F}\times\frac{F}{M}}\Big\rceil^{\frac{1}{2}L}-1\text{ to }1.$$

If now F is put for M, & M is put for F, it will be the probability that the number of males be less than M, but greater than $M-L-1$ to the probability that it be less than

$M-L$ in a ratio greater than

$$\overline{\frac{F+L}{M+L+1}\times\frac{F+1}{F}\times\frac{M}{F}}\Big|^{\frac{1}{2}L}-1 \text{ to } 1.$$

Hence it follows what probability, that the number of males is not greater than $M+L$ nor less than $M-L$ (even if especially those cases are disregarded in which it is able to happen that the number of males is precisely M) to the probability that it may fall outside these limits, in the least has a ratio greater than the lesser of these two quantities

$$\overline{\frac{M+L}{F+L+1}\times\frac{M+1}{F}\times\frac{F}{M}}\Big|^{\frac{1}{2}L} \;\&\; \overline{\frac{F+L}{M+L+1}\times\frac{F+1}{F}\times\frac{M}{F}}\Big|^{\frac{1}{2}L}$$

diminished by unity to unity Q.E.I.

NB. Who will wish to approach nearer the truth, will be able to divide that series $\frac{M+L}{F-L+1}\times\frac{M+L-1}{F-L+2}\times\frac{M+L-2}{F-L+3}\times$&c. into many parts, & to suppose the factors of the individual parts in geometric progression: but we are able to refrain from that more lengthy labor without risk, when the difference will always be exceedingly small between those values of m itself discovered through such diverse suppositions; besides even if especially we should make the value of m itself greater by an actual very small amount, that small amount should be compensated abundantly by means of it which always we have disregarded by a stronger argument, that with any whatsoever it will be clear easily by examining the matter.

It remains that we must apply all these now to the limits observed in that list of 82 years, & we must demonstrate those limits of yours to be more great, as the probability is lesser of the number of infants of each sex for any year whatsoever to be falling between the smaller limits besides. We see from that list the number of males always to be greater than the number of females; hence it is proven always to be a greater facility that a male may be born than a female, and indeed that the calculation be more accurate I have sought the ratio which is between the cases by which a male, & the cases by which a female is able to be born; moreover it is discovered by obtaining the middle among all ratios which of those 82 years the number of males has had to the number of females, as 7237 to 6763; I have reduced the terms of this ratio to the numbers of which the sum makes 14000, because I will suppose the number of infants who are born in each year make 14000. But especially from this middle ratio it has been receded in the Years 1661 & 1703. For in the Year 1661, the number of males to the number of females has had maximum ratio, a ratio namely as 7507 to 6493; but in the year 1703 it has had a minimum ratio truly as 7037 to 6963; the prior numbers differ by 270 units from 7237 & 6763, but the posterior by 200 units. Therefore in order that the argument be with stronger reason let us assume the lesser limit 200, & let us seek how much be the probability, that out of 14000 infants the number of males must not differ by more than two hundreds from 7237. We shall discover (with 7237 substituted for M, 6763 for F, 200 for L)

$$\begin{aligned}
&\frac{1}{2}L \times \overline{\log \frac{M+L}{F+L-1} + \log \frac{M+1}{M} + \log \frac{F}{M}} \\
&\quad = 100 \times \overline{\log \frac{7437}{6564} + \log \frac{7238}{7237} + \log \frac{6763}{7237}} \\
&\quad = 100 \times \overline{0.0542292 + 0.0000600 - 0.0294192} \\
&\quad = 2.4870000,
\end{aligned}$$

of which the number of the Logarithm is as near as possible to $306\frac{9}{10}$. But by substituting into that formula M for F, & F for M, we will discover

$$\begin{aligned}
&\frac{1}{2}L \times \overline{\log \frac{F+L}{M-L-1} + \log \frac{F+1}{F} + \log \frac{M}{F}} \\
&\quad = 100 \times \overline{\log \frac{6963}{7038} + \log \frac{6764}{6763} + \log \frac{7237}{6763}} \\
&\quad = 100 \times \overline{-0.0046529 + 0.0000612 + 0.0294192} \\
&\quad = 2.4830500,
\end{aligned}$$

of which the number of the Logarithm is as near as possible 304. Whence we conclude the probability, that among 14000 infants the number of males must not be greater than 7437, nor less than 7037, to the probability that it must fall outside these limits to have at least the ratio as 303 to 1; and to that point the lot of him who affirms the same will happen with many trials in order, for example, one hundred years successively, will be $= \left.\frac{303}{304}\right\rceil^{100}$ = (as it is discovered by Logarithms) as near as possible $\frac{1000}{1389}$. If I should assume the greater limit 270 observed in the list I must discover a much greater probability, certainly to be able to be put nearly 3 against 1, among 14000 infants the number of males will not be greater than 7507 nor less than 6967 in ten thousand successive years. Therefore it is not a miracle the number of males not to have exceeded this limit for 82 years, nor here some singular Providence of God is operating with anything against the Laws of Nature established and manifested for us by the most wise Creator himself. That which was to be demonstrated.

Addition of the editor: However that made no change at all in the idea to Mr. *'s Gravesande*, one will see the reason for it in the following letter, which he wrote to Mr. *Bernoulli*, in response to that which one just read.

W. 's Gravesande to N. Bernoulli

At The Hague

Sir.

I have received with a sensible pleasure the Letter which you have given me the honor to write to me, & I have read with much satisfaction the writing which you have had the kindness to add to it. I find that you prove very well, & in a manner quite ingenious, all that which you advance. But it seems to me that this does not reverse at all the proof of Mr. *Arbuthnot:* he has even a place in his Letter which makes a judgment that he had thought on that which you prove in your writing, & that he had your sentiment.[1] All that which I find is that you take the word of *Providence* in another sense than he has taken it. I have never pretended to support that God, in regard to the birth of Infants, acts against the laws which he has himself established; this which is the sentiment which you combat with reason in your writing. It is to seek further the battle of words, than to have recourse in order to accord two Mathematicians. But, Sir, before coming to the proof of that which I just advanced, I must say to you that I believe to have had reason when I have said, in the Letter that I have given myself the honor to write you in London, that the number of boys having always been greater during 82 years, it was the same thing as if a token, cast 82 times in sequence, was always to fall back in the same manner. I have well taken care on that which you respond to me: but I saw not at all that this was nothing to the business. I do not doubt at all that you remained in agreement.

In supposing that at the birth of each Infant, there was as much probability for the birth of a boy, as there is for that of a girl, one is able to compare this birth to a token, which is able to fall heads or tails. One will be able also to compare the birth of 1000 Infants, for example, to 1000 tokens. Now it is clear that if one casts 1000 tokens at the same time, there will be as much probability as the greatest number will fall heads, as there will be of probability that the greatest number will fall tails: the number of cases, which give the one & the other, being equal. If therefore someone wagers that 1000 tokens, being cast 82 times in sequence, will fall back always in a way that the greatest number will fall tails, he wagers that one thing which has $\frac{1}{2}$ probability will arrive 82 times in sequence: now in wagering that one token, cast the same number of times, will fall always tails, he makes the same wager. Therefore I have had reason to say that since the number of boys has always been greater during 82 years, this was the same thing as if one token, cast 82 consecutive times, was always fallen back in the same manner. *This which it was necessary to demonstrate.*

I must again add here, Sir, that I believe you to have said, when I have had the honor to see you in The Hague, that I did not take the argument, of which there is concern, on the equality of the number of the boys & of the girls: I believe likewise, if I myself remember well, to have added that that which surprised the most, it was that the numbers of the two sexes were not approximate of odds, that they were not made according to the Table; so much I was persuaded of that which you said, that in a great number of tokens, which would be cast at the same time, the number of those which would fall tails, would be nearly always equal to the other.

I come at present, Sir, to that which I have said at the beginning; & here is how I believe that one can prove Providence by that which is arrived at London, according to the Table of baptised Infants.

If chance conducts the world, there is at each birth as much probability that there will be born a boy, as there is that there will be born a girl: chance is not discerning enough in order to put more probability on one side than the other, according as the

[1]It is necessary to avow, said Mr. *Arbuthnoth*, that the equality of the males & of the females is not mathematical, but physical, that which changes much my calculation.

necessity of human kind is able to require it. On this principle we see how much there is to wager against one, that that which is arrived must not arrive.

I begin by setting into a sum the numbers of all the Infants in the Table. I take the 82nd part, which is 11429, which I regard as a mean number; & I suppose that each year there is born this number of Infants in London. In regard to the boys & girls I suppose that the numbers of them are each year in the same proportion as the numbers of the Table, according to what the number of boys in 1661 had been 6128, & that of the girls 5301, & in 1703 the one of the boys 5745, & the one of the girls 5684: these are the two years, as you have also remarked, in which the number of the two sexes have differed the most & the least. I propose to present this Problem:

A wagers against B that among 11429 tokens, cast at the same time, the number of those which will fall tails will not be more than 6128, & not less than 5745, but that it will be between these two numbers. I demand the lot of A & of B.

I have made the calculation of it, & I have found that when the lot of A is 1, the one of B is 2 & a fraction of which I myself do not remember, & I have none of my papers here. After having resolved this problem, I propose further this one:

A wagers next against B, that if one casts 82 times in sequence the tokens, of which one just spoke, that which he comes to wager will arrive all the time. One demands the lot of A & of B.

I have further made the calculation, & I have found that when the lot of A is 1, the lot of B is expressed by a number of 44 digits; this which shows clearly that if the world is led by chance, there is a number of 44 digits against 1, that that which is arrived must not arrive: whence one must conclude that this is an intelligent Being who has directed the birth of the Infants, & not a blind chance. *This which it was necessary to demonstrate*.

But this intelligent Being has been able to produce this extraordinary effect in two ways: either by one particular direction by which God would act against the laws established by himself; this which would be a true miracle: or else by establishing since the beginning one law, through which the birth of boys was more probable than that of girls, & that of a degree necessary in order to produce the effect that we see to arrive every day, & which is most useful to human kind. This last way is the one by which Mr. *Arbuthnot* believes that God is served; he indicates likewise in his Letter the manner in which he believes that God has put the greatest degree of probability, since the formation of the first man, on the side of the boys.

In your writing, Sir, you prove very well that, supposing this greatest degree of probability, that which is arrived has been able to arrive naturally, & this is that which Mr. *Arbuthnot* has supposed without proving it. It seems to me that in order to destroy his argument, it would be necessary to show that the chance can put in the birth of boys a greater degree of probability than in that of girls; & it would be necessary again that chance was able always to conserve this same degree of probability on the side of boys.

You show that by supposing the existence of a God, the birth of Infants can arrive naturally; that is to say, according to the laws that this Being has established. Mr. *Arbuthnot* on the contrary supports, that if chance leads the world, the birth of Infants has not been able to arrive naturally, & consequently this assumption is false.

Here is, Sir, what is my sentiment; I take the liberty to say it to you without any disguise, in praying to you to wish well to write me that which you think on it: I wish strongly of you to see in accord with Mr. *Arbuthnot*. I am &c.

Addition of the editor: This letter made an impression on Mr. *Bernoulli*; one will judge on it by the following extract from the response that he made to Mr. *'s Gravesande* 30 December 1712.

N. Bernoulli to W. 's Gravesande

30 December 1712

You have made me, a sensible pleasure in writing me at length all that which you think on the argument of Mr. *Arbuthot*: I am in accord with all that which you say; but although I have always been of the same sentiment, I believe nonetheless that you take this argument in another sense than Mr. *Arbuthot* has taken it, at least in the writing which he has inserted into the Philosophical Transactions: because he draws his argument principally from the equality of the number of boys & of girls, & not from this that the number of boys surpasses always the one of the girls. He has supposed that the birth of a boy & of a girl is equally possible, & he has believed that supposing this equality of birth, there is little probability that the number of boys & of girls must arrive between some small limits, that is to say that the difference between the boys & the girls is small with respect to the number of all the Infants, this which is the only thing that I refute. It is true that if the birth of a boy & of a girl is equally possible, it is very little probable that the number of boys surpasses the one of the girls many times consecutively: now as this is arrived nevertheless in London, I conclude, & everyone will conclude it also, that in London the birth of a boy is easier than that of a girl; & that this is an effect of Providence of God, this is that which I never denied. I support only here, that Mr. *Arbuthnot* deceives himself in believing that there is a small probability that the numbers of boys & of girls must approach so near. If he had made some observations on the number of males & of females in another country, where the birth of a boy & of a girl is equally easy, (that there is some such places, it is that which I believe to be quite true, because by that which one has said to me, there is in Switzerland certain places where the number of girls surpasses that of boys, where sometimes the number of boys is greatest, sometimes the one of the girls;) he would nonetheless draw from his calculation the same conclusion, that is to say that it would be little credible that the numbers of boys & of girls will approach very nearly, & between some limits such as one will have found them by the observations; this which would be very false, as you will understand it easily. Mr. *Arbuthnot* made his argument to consist in two things; 1°. in this that, supposing an equality of birth between girls & boys, there is little probability that the number of boys & of girls is found in some limits quite near equality: 2°. that there is little probability that the number of boys will surpass a great number of times consecutively the number of girls. It is the first part that I refute, & not the second. If I had had the good fortune to meet you in The Hague, I would have had the occasion to explain myself more amply on this matter, & to end in a few words our controversy, which, by forming the question well, would be first vanishing: but I hope that you will be in accord with that which I just advanced, & I believe to have sufficiently responded to your objections.

Addition of the editor: One sees by this Letter of Mr. *Bernoulli*, that he agrees with that which makes the principal force of the argument of Dr. *Arbuthnot*, it is that the number of boys born, has always surpassed by a certain quantity the number of girls, & it is there precisely that which Mr. *'s Gravesande* has considered in the preceding dissertation, against which, consequently, the objections of Mr. *Bernoulli* carry not at all.

We add to this that Dr. *Arbuthnot*, as Mr. *Bernoulli* has remarked, supposes first, it is true, a perfect equality between the numbers of boys & of girls who are born; an assumption which renders his calculation very just, of the same assent as Mr. Bernoulli, after which he observes that this equality not being mathematical, his calculation is changed that way, & is proved by becoming more feeble; but in order to render to him all its force, he passes to the examination of the probability that there is that there must be born constantly more males than females; & it is from the infinite smallness of this probability, that he draws principally his proof in favor of Providence.

It is true that Mr. *Bernoulli* tries to diminish it, by remarking that he can have some places where there are born fewer male Infants than females, for example in some quarters of Switzerland, where the number of girls carry it away over that of the boys. But without entering into the examination of this remark, which would suppose some observations which we do not have; I would wish that he had not cited for example the Swiss, where he says, that in some places *there are*, not that there *are born*, more girls than boys. He knew well that in this country many of these last quit their Fatherland, in order to go to serve in the country of strangers. But by supposing the truth of the fact, it would follow always that the City of London is more favored by Providence, than the other parts of the Earth; this which Mr. Bernoulli would not have, without doubt, wished to avow.