CALCULATION OF CHANCES

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In the year 1685 MR. JACQUES BERNOULLI has proposed the following two problems in the Journal des Sçavans.

Sixth Example[1]

Two players A and B play with two dice. The one will be winner who first will cast a determined number of points. A will make first a coup, next B, next A will make 2 of them and B also; next A 3 and B equally 3 coups, and thus consecutively the one after the other according to an arithmetic progression; or else A will make first one coup, next B 2 coups, next A 3 coups, B 4 coups, etc. until this that one of them will have won the game. What will be the chance of each of them?

In order to resolve the first question, I note that there are an infinite number of players, that A casts first 1 time, B equally 1 time, next C and D each 2 times, next E and F each 3 times; and that they themselves propose each to cast the same number of points. If one has calculated then the part which reverts to the infinite number of players of odd rank, one has the part of A in the case of the first problem. The remainder of unity constitutes the part of B. In the formula which precedes the first Example belonging to the Problem considered, one has then $b = c = d = e = a$. Moreover $r = s = 1$, $t = v = 2$, $w = x = 3$, etc. One finds therefore for the part of A the following infinite progressions

$$\frac{a^1}{a^1} + \frac{a^4}{a^2} + \frac{a^9}{a^3} + \frac{a^{16}}{a^4} + \frac{a^{25}}{a^5} + \text{etc.} - a^1 - a^4 - a^9 - a^{16} - a^{25} - a^{36} - a^{49} - \text{etc.}$$

The part of B is expressed by

$$a^1 + a^4 + a^9 + a^{16} + a^{25} + a^{36} + \text{etc.} - \frac{a^4}{a^2} - \frac{a^9}{a^3} - \frac{a^{16}}{a^4} - \frac{a^{25}}{a^5} - \text{etc.}$$

In the case of the second equation the part of A is

$$1 + a^3 + a^{10} + a^{21} + a^{36} + \text{etc.} - a - a^6 - a^{15} - a^{28} - \text{etc.}$$

One sees easily how one is able to continue these series to infinity: one finds the terms in the following fashion:

$$\begin{array}{cccccl} 1 & a^3 & a^7 & a^{11} & a^{15} & \text{(geom. progr.)} \\ & 1 & a^3 & a^{10} & a^{21} & \\ \hline 1 & +a^3 & +a^{10} & +a^{21} & +a^{36}, & \text{etc.} \end{array}$$

and

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[1] Calculation of chances by means of algebra. pp. 63–64 of the edition of Vollgraff.

$$\begin{array}{ccccc} a & a^5 & a^9 & a^{13} & a^{17} \\ & a & a^6 & a^{15} & a^{28} \\ \hline -a & -a^6 & -a^{15} & -a^{28} & -a^{45}, \end{array}$$

(geom. progr.) etc.

The rest of unity constitutes the part of B.