SUPPUTATION DES AVANTAGES DU BANQUIER DANS LE IEU DE LA BASSETE

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Jean de Préchac wrote *La Noble Vénitienne ou la Bassette, Histoire galante*, a Roman allegory, published by Claude Barbin in 1679. In this he gave an explanation of the game of Bassette in the first ten pages. The *Journal des Sçavans* presented comments on this work on pages 37–38. After this short description of the book, the summary of the work of Sauveur follows.

THE NOBLE VENETIAN OR BASSETTE
Histoire galante. *In* 12. In Paris at the house of Claude Barbin 1679.

The game of Bassette has made so much noise this winter from the attachment with which one has played it at the court that there are few folk who are not aware presently that which it is. The history of it is not so well known. This is that which this author teaches us in this little Book which contains only a pure allegory, of which one finds the Key at the end & the rules of the game at the beginning of the work. As one claims that it is a noble Venetian who has invented Bassette, this author has given to it the name of Noble Venetian, & under this title he describes how this game has been banned from Venice a little after which it had been known, how it has been received next in the rest of Italy, & how finally M. Justiniani Ambassador for the Republic of Venice close to His Majesty has introduced it into France 4 or 5 years ago, with all that which has happened of the pleasant thereupon. That which is of another nature.

COMPUTATION OF THE ADVANTAGES OF THE
Banker in the game of Bassette

The vogue which this game has had for some time having been to make many observations by so many persons, the mathematician M. Sauveur has made at the beginning of last November by the rules of algebra, this computation which seems to have given occasion to make the belief that there are some throws only for the players. Those who give themselves the effort to read it will see that he claims only that there are some throws less disadvantageous to the players than others, & they will notice that he supposes always an equality of fortune that one good fortune or one misfortune is able to change. There is place to believe that he is not being deceived in the computation of the tables which he gives, because the first three correspond to three of those rules that Mr. Huygens has taken the pain to calculate, & of which he has given the foundation as well as some others which regard the games of chance, in the Treatise which he has published formerly on these matters.

Date: Journal des Sçavans, Lundy 13 Fêvrier 1679.
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In order to better enter into this computation it is necessary to give first an idea of the most essential of this game, according to the manner on which one has based the following reasonings. Italics marks precisely that which regards the game.

The one who deals who one calls the Banker *or* Dealer *has an entire deck of fifty-two Cards, & those who play against him have each in hand thirteen cards of one color,* which one calls the book. *After the dealer has shuffled his cards, the players reveal in front of him such cards in their book as they wish, on which they lay money at discretion; next the dealer turns his deck of cards, in such a way that he sees first what was on the bottom. After this he draws his cards two by two until the end of the deck by beginning with that which he sees; & by the nature of this game the first of each pair or hand, is always for him, & the second* ordinarily *for the Player, in such a way that if the first is for example a King, the Banker wins all that which has been laid on the Kings, but if the second is a King, the Banker gives to the Players as much as they have laid on the Kings,* & in precisely this the advantage of the Banker is not greater than that of the Player. But it is necessary to note.

1. That if *the first & the second cards are, for example some Kings, (that which one calls* doubles) the turn must be null, however *the Banker wins what has been laid on the Kings*.

2. *Each Player has the liberty of laying some money on such card as he wishes when the game is begun*, in such way that if he laid money, for example on a Queen when the game is begun, it may happen that in the rest of the cards, there were 4, or 3, or 2 or finally 1 Queen: this which diversifies the advantage of the Banker. *But it is necessary to note that the pair or the hand of which he sees the first or the second when he lays is null in regard to the card on which he just laid; & if the card on which one has laid is encountered in the second of the hand which is null, the game is ended for that card, this is why it is necessary to lay anew, but if it is not encountered there the Banker faces in the first of the following hand when he wins, that is to say that he takes only two thirds of that which is laid on the card*.

3. But when there remains no more than one similar card to that on which one has laid, the Banker no longer has advantage, because he is no longer able to have doubles; this is why *the last card is null*, which must be for the player, in such way in order to make that the Banker has always the advantage.

4. However because the advantage of the Banker would be too great, one diminishes it by making that *when he wins on the first hand in which he is able to win a revealed card, he faces at the time, that is to say that he takes only two-thirds of that which has been laid on this card,* in such way that he loses one-third.

It is easy to see 1. That if the first of each pair was for the Banker, the second was always for the Player, the advantage would be equal to one another. 2. That the advantages of the Banker are the doubles & the last of all the cards which is null. That the faces diminish the advantage of the Banker because he loses one-third. That if one lays when one sees the first card of a hand, the second which is null diminishes the advantage of the Banker, as one will see in the following.

In order to judge in general of the greatest & of the least advantage of the Banker. Note 1. that this advantage is so much greater when doubles can happen more easily. Now it can happen more easily, for example with a King, when in a similar number of cards there are 4 Kings than when there are in them only 3 or 2 or 1, & in this last case there can never be doubles. This is why the advantage of the Banker is greater when in a certain number of cards there remain 4 Kings, than when there remain 3 of them & further in this case than if there remain two, & there would be more advantage when there remains two Kings than when there remains only one of them, if this King were not null when it is encountered in the last of all the cards, this which augments more the advantage of the Banker than if there remained 3 Kings, but less than if 4 of them remained, as it seems by the exact calculations that one has made on them.

2. The advantage of the Banker grows in measure as the number of cards decreases, because if there remained, for example no more than 10 cards, in which there were 4 Kings, doublets would be formed more easily than if the same 4 Kings were found in 40 cards; likewise if there were one King in 10 cards, the Banker would have the 5 first cards in each hand for himself, & the Player would have only 4, since the last is null, that which would give cause that in 10 cards the Banker would have the advantage of one in ten, instead that if there would remain 40 there would be only one in 40. Whence it follows that the greatest advantage of the Banker is when one lays at the end of the game on one card of which there remains yet 4 similar, & that the least advantage is when one lays at the beginning of the game on one card of which there remains no more than two of them.

3. When one lays on one card when one sees only the first of a hand, the second of that hand being too *Jeune*, that is to say null, diminishes a little the advantage of the Banker, because if the card similar to that on which one has laid is encountered the deal would be null, & the game would end for that card, & then the Banker would miss having the advantage which he would have if this card would not be encountered at all.

In order to judge in particular of the advantage of the Banker one has considered first this advantage in a single hand, then in 2, then in 3, in 4, &c. supposing that there is encountered in this hand presently one card, presently 2, 3, & then 4 similar cards to that on which one player has laid, without having considered first the faces. One has made a table of it, of which the usage is to know in what manner the advantages of the Banker increase when the face is past, & because he has considered the advantages & the disadvantages of the Banker in all ways possible, one has made many Tables of which each contains some 100 different cases, & one has reduced them to decimal parts in ratio to 100000.

In the end to compute more easily all these different cases one has reduced them to the general formulas expressed by the six Tables preceding, by calling n the number of hands which remain to the player.

In order to understand the usage of the Tables, suppose that one has laid a certain sum of money on a King, when there remains no more than 3 of them in 8 hands, to relate through that of which one sees first which is consequently null, it is found in Table IV (which satisfies these conditions) the formula $\frac{nn-2n}{4nnn-12nn+11n-3}$ in column 3 which is that which it is necessary to take, when there remain no more than 3 Kings. It is necessary to suppose that n signifies 8 hands which remain to the player, as when nn will signify 8 times 8 or 64 & nnn 64 times 8 or 512.

n	signifies	8
nn		64
nnn		512

$nn-2n$, means 64 less two times 8 or 16, that is to say 48. And $4nnn-12nn+11n-3$ signifies 4 times 512 less 12 times 64, plus 11 times 8 less 3, that is to say $2048-768+88-3$ which is 1365, so that instead of the formula $\frac{nn-2n}{4nnn-12nn+11n-3}$ we will have $\frac{48}{1365}$, which means that on 1365 the Banker has 48 of advantage on the Player. Or if one wishes to reduce it into decimal parts, it is necessary to multiply 48 by 100000 in order to have 4800000 which it is necessary to divide by 1365, the quotient or the exponent 3.516

Tables	Numbers of Kings which remain in the cards to play	1	2	3	4
I	Advantages of the Banker in the hands which remain to play when the first not faces.	$\frac{1}{2n}$	$\frac{1}{2n-1}$	$\frac{3}{4n-2}$	$\frac{4n-5}{4nn-8n+3}$
II	If the first is that which faces it is necessary to take off for the face.	$\frac{1}{6n}$	$\frac{1}{3n}$	$\frac{1}{2n}$	$\frac{2}{3n}$
III	Remainder for the advantage of the Banker in the hands of which 1. is a face.	$\frac{1}{3n}$	$\frac{n+1}{6nn-3n}$	$\frac{n+1}{4nn-2n}$	$\frac{4nn+n-6}{12nnn-24nn+9n}$
IV	Advantages of the Banker in the hands which remain by beginning with that which is null.	$\frac{2}{6n-3}$	$\frac{n}{6nn-9n+3}$	$\frac{nn-2n}{4nnn-12nn+11n-3}$	$\frac{4nn-7n-3}{12nnn-36nn+33n-9}$
V	Advantages of the Banker in each hand which does not face without having regard to the following.	0	$\frac{1}{2nn-n}$	$\frac{3}{2nn-n}$	$\frac{6}{2nn-n}$
VI	Disadvantages of the Banker in the hand which faces without having regard to the advantage which he has in the following.	$\frac{-1}{6n}$	$\frac{-2n+7}{12nn-6n}$	$\frac{-2n+19}{12nn-6n}$	$\frac{-2n+37}{12nn-6n}$

indicates to us that the advantage of the Banker is greater than 3 on 100, or of 3516 on 100000. One can make the same thing on the other cases.