Pratica
D'ARITHMETICA E GEOMETRIA*

Lorenzo Forestani da Pescia (?-1623)

Libro Quinto, pp. 364–367, Edition of 1682

Introduction

The first edition of this work of Lorenzo Forestani was published in 1602 at Venice. Here we present the relevant text[1] from the edition of 1682 of the *Pratica* [1] associated with the division of stakes and a translation of it into English. A scan of this work is available through Google Books.

However, the headings are taken from the table of contents, not from the body of the work where there is no explicit indication that a new topic is begun.

Forestani is interesting for the colorful settings in which he places his problems. There is nothing original here. Forestani comments on the errors of Pacioli and of Peverone. He further critiques Francesco Pagani, the author of another arithmetic published in 1591.

D'alcuni che giuocano alla palla, a balestrare, e d'altre variete domande.

Un Gentihuomo già vacchio, ritrovandosi a una sua villa, e dilettandosi grandemente del giuoco di palla, chiamò due giovani Contadini, e disse, eccovi 4 ducati, giocateli qui in mia presenza alla palla, e chi di voi prima vince 8 giuochi, voglio, che habbia vinto li 4 ducati, e cosi cominciarono a giocare, e quando un di

Questions of those who play with the ball, with crossbow, and other varieties.

An already old Gentleman, finding himself again at his villa, and taking great delight in the game of ball, called two young Laborers, and said, "Here you have 4 ducats to play in my presence with the ball here. And who of you first gains 8 games, I wish that he has won the 4 ducats." And therefore they began to

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[1]The original text has been reproduced very closely. It must be admitted that the printed text is very difficult to discern in many places and consequently errors are quite possible. Some small liberties have been taken in order to make it more readable. The long "s" is replaced everywhere by "s." The contraction suppressing the letter "m" or "n" as, for example, in "adũque" for "adunque" is expanded. At that time the letter "u" represented the letter "v" when it appeared within a word and vice versa when the letter "u" began a word. In most cases, but not all, the more appropriate letter has been used as, for example, with the substitution of "deve" for "deue." Spelling has not been modernized. Thus "hauendone" will appear as "havendone" but not "avendone."

loro hebbe vinto 6 giuochi,[1] e l'altro 3 si perse la palla, e non poterono finire, & gentiluomo disse, eccovi i denari, dividete li fra voi, si domanda quanti ne toccarà per uno.

play. And when one of them had gained 6 games, and the other 3 they lost the ball, and could not end, & the gentleman said, "Here you have the money, divide it between you." I ask how much of it each one receives.

Nel risolvere simili propositioni son diverse l'openioni, però questa a noi pare la più retta, e la più commune, e prima diremo così, che quello il quale vorrà li 4 ducati, bisognarà che vinca 8 giuochi, e l'altro non ne puol vincer più che 7. perciòche fra di loro non può correre più di 15 giuochi; La onde il primo vincendo 5 giuochi, viene a vincere $\frac{5}{15}$ cioè $\frac{1}{3}$ de' 4 duc. & il secondo che vince 3 giuochi, viene a vincere $\frac{3}{15}$ cioè $\frac{1}{5}$ de'detti 4 duc. di maniera che tra il primo, e secondo vengono a vincere $\frac{8}{15}$ de' 4 duc. per la qual cosa chiaramente si conosce, che vi resta $\frac{7}{15}$ i quali non sono affaticari, nè giuocati, nè vinti da nessun di loro, e perciò bisogna divider li per metà, di $\frac{7}{15}$ piglia adunque la metà, che'è $\frac{7}{30}$ & aggiungili a $\frac{1}{3}$ fanno $\frac{17}{30}$ e tal parte ne tocca al primo, e l'altra metà, cioè $\frac{7}{30}$ aggiungili a $\frac{1}{5}$ fanno $\frac{13}{30}$ e tal parte ne tocca al secondo. Hora dividasi 4 ducati per modum societatis, dicendo cosi, il primo deue trarre per 17 & il secondo per 13 de'detti 4 ducati, opera, trovarai che il primo doveva havere ducati $2\frac{4}{15}$ & il secondo duc. $1\frac{11}{15}$ e questa è la vera solutione di simili proposte.

In resolving various similar propositions the opinions are diverse, but this to us seems most correct, and most common. And first we will say therefore, that that one who will ask for 4 ducats, that he will have need to win 8 games; and the other is not able to win any more than 7. Therefore between them it cannot run more than 15 games; so that the first one, winning 5 games, comes to gain $\frac{5}{15}$, that is $\frac{1}{3}$ of the 4 ducats & the second, who wins 3 games, comes to gain $\frac{3}{15}$, that is $\frac{1}{5}$ of the said 4 ducats in a way that between the first one, and the second, they come to gain $\frac{8}{15}$ of the 4 ducats. For which thing clearly it is known, that $\frac{7}{15}$ remain to you which is not exhausted, neither by games, nor by any of them winning. And therefore you must divide it in half. Then seize the half from $\frac{7}{15}$, that is $\frac{7}{30}$; & you add it to $\frac{1}{3}$. It makes $\frac{17}{30}$ and such part the first one receives; and the other half, that is $\frac{7}{30}$, you add to $\frac{1}{5}$. It makes $\frac{13}{30}$ and such part the second receives. Now divide 4 ducats in the societal way, saying therefore, the first one must receive for 17 & the second for 13 of the said 4 ducats. Work. You will find that the first one must have $2\frac{4}{15}$ ducats & the second $1\frac{11}{15}$ ducats and this is the true solution of similar proposals.

Tre soldati essendo dentro ad una Fortezza per la quale andando a spasso, travorono uno scudo, e chiascun di loro lo voleva pur alla fine s'accordorono che si dovesse giocare alle pallottole, con patto, che chi di loro vincerà prima 14 giochi habbia vinto le scudo, cioè lire 7 accadde che quando il primo hebbe vinto 10

Three soldiers being inside a Fortress, by going for a walk, found a scudo, and each of them wanted also in the end to be agreed that it had to be played with bullets, with the pact, that who of them first will win 14 games has gained the scudo, that is 7 lira. It happened that when the first one had won 10 games, the second 8 giuochi, il secondo 8 & il terzo 5 gli convenne andare in guardia, si domanda in che modo farà dovere che dividino il detto scudo, e che parte ne toccherà a ciascuno.

Fa cosi, prima vedi, quanti giuochi posson fare al più fra tutti tre, chiara cosa è che un di lore bisognava che vincesse 14 giuochi a voler vincer lo scudo, e gli altri due puol'esser che vinchino 13 giuochi per uno, adunque il più che posson fare fra tutti tre sono 40 giuochi, de'quali il primo havendone vinti 10 sono $\frac{1}{4}$ di tutti; & il secondo havendone vinti 8 sono $\frac{1}{5}$ & il terzo havendone vinti 5 sono $\frac{1}{8}$ che giunte insieme queste tre parti fanno $\frac{23}{40}$ e tal parte dello scudo hanno vinto fra tutti tre, del quale gli mancò loro il tempo da vincere il resto cioè $\frac{17}{40}$ e questa parte non essendosi ancora giocata, nè vinta, & affaticara da nissun di loro, il dovere vuole che si divida vgualmente per terzo, e perciò prendi il $\frac{1}{3}$ di $\frac{17}{40}$ ne viene $\frac{17}{120}$ e tal parte devi aggiungere alla prima parte data di sopra a ciascun di loro; la qual'aggiunta che l'havrai, troverai che al primo ne li toccherà $\frac{47}{120}$ al secondo $\frac{41}{120}$ & al terzo $\frac{32}{120}$ e se vuoi sapere quante lire toccherà a ciascuno, vedi separatamente ciascuna delle sopradette parti quante lire sono, multiplica $\frac{47}{120}$ via lire 7 fanno lire 2.14.10 e tante lire tocco al primo, & al secondo gli toccò lire 2.7.10 & al terzo lire 1.17.4 che in tutto sono lire 7 e questa ci pare che sia la vera solutione, e non quella come vuol Fra Luca nella proposta che fa del giuoco della palla, e nel giuoco di trar con la balestra, la qual propone cosi, e dice.

& the third 5 they agree to go on guard. It is asked in what way that scudo must be divided, and the part each of them will receive.

One makes therefore, first you see, how many games at most one is able to make among all the three, it is clear that of them one had to win 14 games to wish to gain the scudo, and the other two are able to win 13 games each, therefore the most that they are able to make among all three are 40 games, of which the first one having won 10 has $\frac{1}{4}$ of all; & the second having won 8 has $\frac{1}{5}$ & the third having won 5 has $\frac{1}{8}$. That added together these three parts make $\frac{23}{40}$; and such part of the scudo they have gained among all three. What remainder they lacked the time to win, that is $\frac{17}{40}$, and this part not being itself yet played, nor won, & exhausted by none of them, it must be wished that it be divided equally by three, and therefore take $\frac{1}{3}$ of $\frac{17}{40}$ of it comes $\frac{17}{120}$ and such part you must add first to the part given above to each of them. Adding these you will have it. You will find that first of them will receive $\frac{47}{120}$, to the second $\frac{41}{120}$ & to third $\frac{32}{120}$. And if you want to know how many liras each will receive, see separately how many lira are each of the aforesaid parts. Multiply $\frac{47}{120}$ by 7 lira makes 2.14.10 lira[2] and the first receives so many lira, & to the second 2.7.10 lira & to third 1.17.4 lira. That in all there are 7 liras and this seems to us that it is the true solution, and not that one like Fra Luca wants in the proposal that he makes of the game of the ball, and in the game to draw with crossbow, which he proposes therefore, and says.

Error di Fra Luca, e del Peverone

Son tre giovani i quali fanno a balestrare a chi prima di loro fa 6 colpi ineglio quello habbia da vincer 10 ducati, e quando il

Error of Fra Luca, and of Peverone

There are three young people who play with a crossbow; to the first who makes 6 points, that one has 10 ducats primo hebbe fatto 4 colpi, il secondo 3 & il terzo 2 non voglion far più, e d'accordo voglion partir la proposta, si domanda quanti ne toccherà per uno. Il detto Fra Luca dice cosi, che il piu chè posson fare fra tutti tre sono 16 colpi, perche puol'essere che tutti tre sieno a 5 colpi, & uno poi se ne farà per haverne 6 adunque de 16 colpi che posson fare in tutto il primo n'ha 4 che sono $\frac{1}{4}$ di tutti, il secondo, n'ha 3 che sono $\frac{3}{16}$ & il terzo n'ha 2 che sono $\frac{1}{8}$ dipoi egli divide 10 ducati in questo modo cioè, al primo ne dà $\frac{1}{4}$ che sono duc. $2\frac{1}{2}$ al secondo ne da $\frac{3}{16}$ che sono duc. $1\frac{7}{8}$ & al terzo ne da $\frac{1}{8}$ che sono duc. $1\frac{1}{4}$ i quali sommati tutti insieme fanno duc. $5\frac{5}{8}$ e questi gli trahe di duc. 10 restano duc. $4\frac{3}{8}$ li quali divide per modo di Compagnia dicendo, il primo mette 4 cioè per 9 giuochi[3] vinci, & il terzo metti 3 & il secondo mette 2 & hanno a dividere duc. $4\frac{3}{8}$ de' quali al primo ne tocca duc. $1\frac{17}{18}$ e questi aggiungi con duc. $2\frac{1}{2}$ che di sopra gli toccorono, fanno duc. $4\frac{4}{9}$ e tanti ne vuol che ne tocchi al primo; ha qual cosa poteva fare in una volta sola dicendo. Il primo deve trar per 4 il secondo per 3 & il terzo per 2 & hanno a partire duc. 10 de' quali al primo toccherebbe duc. $4\frac{4}{9}$ come di sopra s'è detto.

Ma noi non approviamo questa sua openione, e conclusione, perciò che li duc. $4\frac{3}{8}$. i quali non si son vinci da nissun di loro, esse gli divide per rata de colpi fatti i quali non vi hanno parte alcuna, atteso che i colpi fatti meritano una parte de'detti denari, e l'altra parte che non s'è affaticata da nessun di loro, non è dover divider la secondo la rata de' primi colpi, ma si bene

to gain, and when the first one had made 4 points, the second 3 & third 2, they do not want to make more. And in agreement they want to leave the proposal. It is asked how much of it one will receive. Fra Luca says therefore, that the most they are able to make among all three are 16 points, because to be able to be that all three may have 5 points, & then one will be made in order to have 6. Therefore of the 16 points that are able to be made in all, the first has 4, these are $\frac{1}{4}$ of all. The second has 3, these are $\frac{3}{16}$, & third has 2 these are $\frac{1}{8}$. Then one divides 10 ducats in this way: that is, to the first give $\frac{1}{4}$ these are $2\frac{1}{2}$ ducats. To the second of them give $\frac{3}{16}$, these are $1\frac{7}{8}$ ducats. & to the third of them give $\frac{1}{8}$, these are $1\frac{1}{4}$ ducats. Which summed all together make $5\frac{5}{8}$ ducats. And these subtracted from the 10 ducats, there remain $4\frac{3}{8}$ ducats, which you divide by way of a Company saying: the first one you put 4, that is, for 9 games won, & the second[4] you put 3 & the third[5] you put 2 & you have to divide $4\frac{3}{8}$ ducats of which the first one of them receives $1\frac{17}{18}$ ducats. And these you add with $2\frac{1}{2}$ ducats received above, it makes $4\frac{4}{9}$ ducats and so much of it he wishes that the first one receives; he has what thing one could make at one time a single statement. The first one must obtain for 4 the second for 3 & the third for 2 & they have 10 ducats to start of which the first one would receive $4\frac{4}{9}$ ducats as he himself said above.

But we do not approve of this his opinion, and the conclusion, therefore that the $4\frac{3}{8}$ ducats, which have been gained by none of them, to be divided pro rata with the points made, which do not have to you some part. It is expected that the points made deserve a part of said money, and the other part that is exhausted by none of them, must not be divided according to divider la vgualmente per terzo, in questo modo cioè.

Noi habbiamo detto che il primo havendo vinto 4 giuochi, ò vero colpi, viene ad haver vinto il $\frac{1}{4}$ de' tutti i colpi che si potevon fare, cioè il $\frac{1}{4}$ di 16 e cosi viene a vincere il $\frac{1}{4}$ di tutto quel che giuocano, & il secondo ne viene a vincere $\frac{3}{16}$ & il terzo $\frac{1}{8}$ che giunte insieme tutte queste parti, fanno $\frac{9}{16}$ e tal parte vengono ad haver vinto fra tutti tre de' detti 10 ducati, & il resto cioè $\frac{7}{16}$ non essendo vinti da nessun di loro, perciò convien dividere $\frac{7}{16}$ vgualmente in tre parti, che ne viene $\frac{7}{48}$ per parte i quali aggiunti con $\frac{1}{4}$ che n'haveva vinto il primo per li 4 colpi fatti, fanno $\frac{19}{48}$ e tal parte doverebbe havere il primo de' detti 10 ducati.

Aggiungi per il secondo, e per il terzo, nel medesimo modo, troverai, che al secondo ne toccherebbe il $\frac{1}{3}$ & al terzo ne toccherebbe $\frac{13}{48}$.[6] Piglia adunque per il primo $\frac{19}{48}$ di 10 ducati, ne viene ducati $3\frac{23}{24}$ e tanti li dice che ne toccherebbe al primo, & al secondo, gli toccherebbe duc. $3\frac{1}{3}$ & al terzo duc. $2\frac{17}{24}$ i quali sommati insieme, fanno ducati 10 e questa teniamo che sia la vera solutione, e non openione, e dica altri ciò che vuole.

Due giocano a 10 partite, ò vero 10 giuochi, & il primo n'ha guadagnate 7 e l'altro 9 accade certo inconveniente, che non posson finire, se vuoi sapere quel che ciascuno doverà havere del deposito fa cosi, diffalca 7 da 10 resta 3 similmente diffalca 9 di 10 resta 1 la progression continua di 3 è 6 è quella d 1 è 1 partendo adunque il deposito in 7 parti, 6 toccano al secondo, & 1 parte al primo.

Questa la propone, e dispone Gio. Francesco Peverone nel modo sopradetto.

the rate of first points, but to divide themselves well equally by the third, that is in this way.

We have said that the first one having won 4 games, indeed points, he comes to have gained $\frac{1}{4}$ of all the points that they themselves are able to make, that is $\frac{1}{4}$ of 16, and therefore he comes to gain $\frac{1}{4}$ of all those in play; & the second of them comes to win $\frac{3}{16}$ & third $\frac{1}{8}$. That adding altogether all these parts, makes $\frac{9}{16}$ and such part come to have gained among all three of the said 10 ducats, & the rest, that is $\frac{7}{16}$, being won by none of them, therefore agree to divide $\frac{7}{16}$ into three equal parts. Then comes $\frac{7}{48}$ for each, which added with $\frac{1}{4}$ that the first of them had won with 4 points, makes $\frac{19}{42}$ and such part the first one must have of the said 10 ducats.

You add for the second, and for the third party, in the same way. You will find, that the second of them would receive $\frac{1}{3}$ & the third would receive $\frac{13}{48}$. Remove therefore for the first $\frac{19}{48}$ from 10 ducats; there comes $3\frac{23}{24}$ ducats and so much he says that the first one of them would receive, & the second, he would receive $3\frac{1}{3}$ ducats, & the third $2\frac{17}{24}$ ducats. Which added together, they make 10 ducats. And this we hold that it is the true solution, and not opinion, and it tells others what it wants.

Two play to 10 matches, indeed 10 games, & the first one of them has earned 7 and the other 9. A certain inconvenience happens, that they cannot end. If you want to know those that each must have of the deposit, make therefore: deduct 7 from 10 there remains 3, similarly deducts 9 from 10 remains 1. The continued progression from 3 is 6 and that of 1 is 1 dividing therefore the deposit into 7 parts, the second receives 6, & the first one 1 part.

This Gio. Francisco Peverone proposes, and arranges it in the aforesaid way e noi diciamo, che al primo toccherà $\frac{17}{38}$ del deposito, & secondo $\frac{21}{38}$, si come per le sopradette ragioni habbiamo dimestrato.[7]

and we say, that the first one will receive $\frac{17}{38}$ of the deposit, & the second $\frac{21}{38}$, as for the aforesaid reasons we have demonstrated.

Openion falsa del Pagani

Quattro compagni giuocano alla palla a 60 il giuoco, a 15 per caccia, a due per banda, e giocando, quando che l'una parte hebbe 45 e l'altra hebbe 15, occorse che si perse la palla, e non poteron finire il giuoco, & andava di scommessa 8 scudi, domando quanto toccherà per parte di detti denari.

Questa è la 23 proposione descrit a dal Pagani da Bagnacavallo nel suo trattato delle due false positioni. La solutione che esso gli da è questa cioè, dice che la maggior quantità di cacce, che fra tutte due le parti si posson fare sono 7 di maniera che la parte che ha 45 viene havere $\frac{3}{7}$ delle dette 7 cacce, e similmente la parte che ha 15 viene havere $\frac{1}{7}$ delle dette cacce, dipoi egli somma insieme $\frac{3}{7}$ con $\frac{1}{7}$ fanno $\frac{4}{7}$ fatto questo procede per via di compagnia dicendo, se $\frac{4}{7}$ quadagnano scudi 8 quanto doverà guadganare $\frac{3}{7}$ della prima parte, & $\frac{1}{7}$ della seconda? e cosi conclude, che alla prima parte toccherà scudi 6 & all'altra parte scudi due, e doppo questa conclusione n'adduce due altre in suo favore, ma pur simili alla sopradetta, le quali per brevità non vogliamo perder tempo a descriver le, per essere tutte openioni erronee.

Ma perche del detto Pagani nell'ottava sua propositione, poco indietro alla sua sopradetta ha descritto, che due Giovani giuocando al Tavoliero 18 ducati (i quali insieme havevon trovati a caso in una borsa) con patto, che gli dovesse havere chi di loro vinceva prima 8 giuochi, e questa la conclude diversamente alla sopradetta, e perciò da noi si adduce questa sua contraditione.

False opinion of Pagani

Four companions play at ball with 60 the game, at 15 per game, with two per group, and playing, when that one party had 45 and the other had 15 it happened that they lost the ball, and they are not able to end the game, & working with an 8 scudi bet, I ask how much each party will receive of the said money.

This is Proposition 23 described by Pagani da Bagnacavallo in his treatise on the two false positions. The solution that he gives is this, that is, he says that the greater quantity of games, that between both parties are able to be made are 7 in a way that the party that has 45 comes to have $\frac{3}{7}$ of said 7 games, and similarly the part that has 15 comes to have $\frac{1}{7}$ of the said games. Then he sums together $\frac{3}{7}$ with $\frac{1}{7}$, it make $\frac{4}{7}$, a fact which proceeds by way of company saying: If $\frac{4}{7}$ earns 8 scudo how much must $\frac{3}{7}$ earn of the first party, & $\frac{1}{7}$ of the second one? And therefore he concludes, that the first party will receive 6 scudi & the other party two scudi, and after this conclusion it adduces two others in its favor, but also similar to the aforesaid one, which for brevity we do not want to lose time to describe, for being all erroneous opinions.

But because of Pagani saying in his eighth proposition, he has described a little previous to his aforesaid, that two young men playing at a Table 18 ducats (which together they had found by chance in a bag) with a pact, that that who of them had to have won first 8 games, and this he concludes diversely to the aforesaid, and therefore by us this, its contradiction, is adduced.

Perciò che nella propositione del giuoco del Tavoliero, dice che il primo di loro che vincerà 8 giuochi, vincerà ancora li 18 ducati, e che un di loro vinse 6 giuochi, e l'altro ne vinse 5 dipoi persero i dadi, e non poterono finire, perilche si domanda quanti ducati toccherà per uno, la solutione di questa propositione dice esser questa cioè, che tutti due potevon fare 15 giuochi, e non più, perche quello che havesse vinto li denari bisognava che vincesse 8 giuochi, e l'altro compagno il più che potesse vincere fa riano stati 7 giuochi, onde perche il primo ha vinto 6 giuochi, di ragione viene a vincer $\frac{6}{15}$ de' detti 18 ducati, & il secondo havendo vinto 5 giuochi, viene a vincer $\frac{5}{15}$ de' detti 18 duc. di maniera che fra il primo, e secondo vengono a vincere $\frac{11}{15}$ de' detti denari, sò che vi resta ancora $\frac{4}{15}$ de' detti 18 duc. i quali non sono stati vinti, nè affaticati da nessun di loro, e perciò il dover vuole, che questo resto si divida per metà.

Therefore that in the proposition of the game of the Table, he says that the first one of them that will win 8 games, will yet gain 18 ducats. And that one of them won 6 games, and the other of them won 5, then they lost the dice, and they could not end. For that he demands how many ducats will each receive, the solution of this proposition he says to be this, that is: that the two are able to make 15 games, and no more, because for he who had gained the money it was necessary that he won 8 games, and the other companion the most that he could win it makes they had been 7 games, now because the first one has won 6 games, by reason he comes to gain $\frac{6}{15}$ of the said 18 ducats, & the second having won 5 games, comes to gain $\frac{5}{15}$ of said 18 ducats in a way that between them the first one, and the second come to gain $\frac{11}{15}$ of said money, so that there still remains to you $\frac{4}{15}$ of the said 18 ducats, which have not been won, nor exhausted to you by any of them, and therefore to have wished, that this remainder be divided in half.

Parti adunque $\frac{4}{15}$ per metà, ne viene $\frac{2}{15}$ i quali aggiungi alli $\frac{6}{15}$ del primo fanno $\frac{8}{15}$ e tal parte tocca al primo compagne de' 18 duc. e similmente gli altri $\frac{2}{15}$ li aggiungerai a $\frac{5}{15}$ faranno $\frac{7}{15}$ e tal dirai che tocchi al secondo de' detti 18 duc. perilche piglia $\frac{8}{15}$ di 18 ne viene duc. $9\frac{3}{5}$ per il primo compagno, e similmente per il secondo piglia $\frac{7}{15}$ di 18 ne viene duc. $8\frac{2}{5}$ e tanti ne toccarebbe al secondo.

Divide therefore $\frac{4}{15}$ in half, from it comes $\frac{2}{15}$ which you add to $\frac{6}{15}$ of the first one. They make $\frac{8}{15}$ and such part the first companion receives of the 18 ducats and similarly the others $\frac{2}{15}$ of it you will add them to $\frac{5}{15}$. It will make $\frac{7}{15}$ and such you will say that the second receives of the said 18 ducats. For that seize $\frac{8}{15}$ of 18. There comes from it $9\frac{3}{5}$ ducats for the first companion, and similarly for the second seize $\frac{7}{15}$ of 18. There comes from it $8\frac{2}{5}$ ducats so much of it the second receives.

Hor questa è quella solutione, che il detto Pagani dà alla sopradetta propositione, la qual conclusione è quella che a noi piace, & approviamo per buona. Ma perche le solutioni di simili proposte consistono nelle openione, e l'openioni,

Now this is that solution, that Pagani gives to the aforesaid proposition, which conclusion is what appeals to us, & we approve for good. Yet because the solutions of similar proposals consist in opinion, and opinions, & the opinions being

& i pareri essendo varii, però la scieremo tal giudicio a più savi, & intendenti, perciòche a noi basta haver detto il parer nostro, e dimostrato questa sua contradittione. La ragione che alcuni adducono in contrario è questa cioè, dicono, che chi ha più giuochi, e più vicino al poter finire, e conseguire il tutto, e percio gli si convien tirare di quei danari per rata de' giuochi vinti, e noi diciamo che la Fortuna si può rivoltar presto, e favorir quell'altro a vincer il tutto, si come infinite volte s'è visto, e vedesi, tanto nel giuoco di palla, come in ogn'altro, ma molto più nelle cose di guerra, si come dottamente ne dimostra l'Ariosto in persona di Carlo con questi due versi.

varied, but we will know such judgment to be the more sensible, & intending, therefore that to us it is enough to have said our opinion, and demonstrated to this its contradiction. The reason that some adduce to the contrary is this, that is, they say, that who has the most games, is the most near to being able to end, and to achieve all, and therefore appropriate to draw of that money by rate of games won. And we say that Fortune herself is able to turn soon, and to favor that other to win all, as it was seen, and is seen, an infinite number of times, as much in the ball game, as in any other, but much more in the things of war, one as scholarly of it Ariosto demonstrates in the person of Charlemagne with these two verses.

Cosi Fortuna ad Agramante arrise
Ch' un'altra volta a Carlo assedio mise.

Thus Fortune smiles on Agramant
Who became anew, sets to besieging Charlemagne.[8]

il qual' havendo assediato Agramante, si rivoltò talmente la Fortuna, che Agramante in un'attimo ruppe l'Esercito di Carlo, e nuovamente l'assediò in Parigi.

who having besieged Agramante, Fortune will turn so, that Agramante in a moment broke off the Army of Charlemagne, and newly will besiege him in Paris.

1 This value of 6 games becomes a 5 in the subsequent solution.

2 That is, 2 lira, 14 soldi and 10 denarii. We have at this time 1 lira = 20 soldi and 1 soldo = 12 denarii.

3 Corrected. The original has 14.

4 Original: terzo or third.

5 Original: il secondo or the second.

6 We have for the second player $\frac{3}{16} + \frac{7}{48} = \frac{11}{3}$ and similarly for the third $\frac{2}{16} + \frac{7}{48} = \frac{13}{48}$.

7 Original has in error $\frac{19}{38}$.

8 *Orlando Furioso*, Canto 27, verse 33.

Forestani's Method

Forestani introduces his preferred method of solution to the problem of the division of stakes in his first two examples. He proposes that the stake initially be divided *pro rata* based upon the number of games won and the maximum number of games that could be played. The remaining stake is then divided equally between the players. Hence, the general solution may be expressed as follows:

In the case of two players A and B, let a and b denote the number of games won by each player respectively and let n denote the number of games required to take the stake. It follows that the shares of the stake to players A and B will be respectively

$$\frac{1}{2} + \frac{a-b}{2(2n-1)}, \qquad \frac{1}{2} + \frac{b-a}{2(2n-1)}$$

In the case of three players A, B and C, let a, b, and c denote the number of games won by each player respectively and let n be as before. It follows that the shares of the stakes of players A, B and C will be respectively

$$\frac{1}{3} + \frac{2a-b-c}{3(3n-2)}, \qquad \frac{1}{3} + \frac{2b-a-c}{3(3n-2)}, \qquad \frac{1}{3} + \frac{2c-a-b}{3(3n-2)}$$

Forestani now cites the contest with the crossbow from Pacioli. Here we have three players A, B and C who again have each won a, b and c games respectively. The stake is apportioned as before *pro rata* of the number of games won with respect to the maximum number of games that could be played. The remaining stake Pacioli apportions in the manner of the company, that is, *pro rata* of the number of games won with respect to the number of games played. It follows then that the stake is apportioned respectively as

$$\frac{a}{a+b+c}, \qquad \frac{b}{a+b+c}, \qquad \frac{c}{a+b+c}$$

Kendall [2] has commented on Peverone in which he mistakingly calls the solution one of the "nearest misses in mathematics." Peverone reasons as follows: Suppose our players lack 1 and 3 points respectively. Each player would stake 1 on the next game. The second will stake 2 on the second game because he would win 2 if he won both. Consequently, he will stake 3 on the third game because he would win 3 if he won all three. Thus, the value of the game to the second player is $1 + 2 + 3 = 6$ and to the first player 1. Hence, the total value of the future is 7.

Francesco Pagani is the author of *Arithmetica Prattica Utilissima, artificiosamente ordinata*, published at Ferrara in 1591. Pagani is criticized for being self-contradictory. The first example, which seems to refer to a version of tennis, is solved in the manner of Pacioli or in the manner of a company. It is not clear what he means by double false position in this context since the proportions can be solved trivially, namely as

$$\frac{4/7}{8} = \frac{3/7}{x} = \frac{1/7}{y}$$

where x and y are the shares of the stake to be awarded to the teams.

The second example, that of the dice game, is solved by the method favored by Forestani.

References

[1] Lorenzo Forestani. *Pratica d'arithmetica e geometria*. Siena, 1682.

[2] M. G. Kendall. Studies in the history of probability and statistics II. the beginnings of a probability calculus. *Biometrika*, 43(1/2):1–14, Jun. 1956.