

De duratione media matrimoniorum, pro quacun- que conjugum aetate, aliisque quaestionibus affnibus.*

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§ 1. Since there is a continuous change of living and dying, now first natality then mortality tables have been constructed for very many years in the writings of different peoples, by which finally the laws of variations and vicissitudes, with marvelous success, have been observed and also established; for however much the fate of each and every ordinary man must be entirely uncertain, yet it is not to be denied, that the mean state, for the great number of men collected indiscriminately, may be answered by nearly invariant laws, withersoever that situation may be recorded or wherever the discussion may be concerning the thing. Thus it has been observed, the number of sons born annually regularly to surpass the number of daughters born annually; indeed, what is more marvelous, actually the inequality itself to observe pretty much without change the same proportion to the total number of births in the Region at the same time: but the latter is not able to be observed except in the case of very great numbers, in which of course the uncertainty of a chance lot, of the ratio possessed of the whole, is demonstrated to nearly vanish; even so those proportions for both sexes seem to be little different in different regions. In the more accurate tables enumerations of pathologies are added, whence we understand the innate character of deadly diseases best; now it has been recognized, smallpox alone has killed in these times the twelfth or thirteenth part of any generation, more or less according to the ratio and difference of peoples; it has been recognized equally, disease of the first infancy to destroy nearly three-tenths part of the entire generation within the first year of birth; it has been further observed, because it pertains more closely to our intention, females commonly to delight in longer life than males; we have the table here constructed on this by the Celebrated D. *Wargentin* in Sweden,¹ which confirms this observation excellently; nor is it able to be attributed to a different rule of living, because that more easily observed prerogative of the sex is evident from the beginnings of infancy constantly and remains with regard to that through all life. Concerning the same² within the first year of birth 1623 little sons die and 1438 little daughters from the number of each sex; the mean life itself from the assumed birth with respect to little sons is 24 years 2 months, with respect to the other sex is 28 years with 10 months; 2008 of the first class arrive to the twentieth year, in the other class 2337, henceforth the number of females constantly surpasses the number of living men. Observations of this kind, whether they refer to the political, or to the medical, mostly lack its utility; but that the broader use of them is apparent by far, if only writers consider from each its importance and make use of them correctly.

§ 2. Besides natality and mortality tables, matrimonial tables are accustomed to be constructed also, which serve foremost the interest of the political arrangement to be elucidated and may be more serviceable,

*Concerning the mean duration of marriages, for whatever age of the spouses, and other related questions.

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¹P.W. Wargentin "Mortaliteten i Sverige, i anledning af Tabell-Verket," 1766.

²*Translator's note:* From the Table of Wargentin.

if only they might be composed more accurately, of course it may be chosen for any marriage, that the age of each betrothed may be indicated by increased Arabic numeral and by another Roman numeral, if the husband or wife will have celebrated either the first or second or even third marriages. Thus from the composed matrimonial tables the traditions introduced among diverse peoples and in diverse cities are understood best for those of the thing, which, if they are less appropriate perhaps to political axiomatics, are able to be attacked prudently.

§ 3. Of the aforementioned tables three fundamentals are able to be formed with individual regular observations brought together by great labor, for nothing here supports the uses of a rule; but it is not able to be said, how many new truths by one calculation thenceforth may be able to be elicited. The mortality tables teach us, for the sake of an example, the expected mean duration of life from any age whatsoever besides, certainly not immediately but through consequences in whatever way by themselves you meet each one; but the question, which now we shall explain in detail, concerning the mean duration of marriages is much more laborious and requires many more deductions also when thence the diverse classes, successions and vicissitudes of human kind should be sought, I have estimated the cost by me of doing the work, if I revealed the way to this argument, because it has much common with some specimen,³ I have communicated before these seven years with the Royal Academy of Sciences of Paris, inserted into the *Memoires* of this Academy in the Year 1760, where I have discussed, concerning the mortality by natural smallpox and by healthful inoculation of them, unaccustomed uses of the analyses of a kind toward this thing.

§ 4. When the question is concerning the mean duration of marriages, for a given age of each spouse; it is necessary that first the table should present for a given number of similar marriages, how many of them are going to remain whole after a single year, but the manner by which a table of this kind may be constructed, yet until now is wanted. The mortality tables will show certainly the number of men as also of women surviving after one year; but each number will have been composed out of widows and out of spouses or of them, who even now live in a matrimonial state; but since these numbers are completely unknown, analysis is required, whereby they are able to be determined and this analysis should be derived out of the art of conjecture. I shall begin from the simpler, and in the first place I shall suppose an equal age in each sex, so that all spouses whether men or women have a common age; in addition next I shall put each sex of the same age subject to the same risk of death indiscriminately, so that the observed is to have the thing itself somewhat differently and to treat a more easily observed life of the sexes more secure from death; yet that observation therefore should not be interpreted, just as if in rich cities or in other places females as well as males may die in fewer numbers annually, since if they are born in the same number, it is necessary as well that they die in equal number; while it must be observed, in every city more females to survive than males, whence it follows the sum annually of the dying for each sex certainly to be able to be absolutely equal, but it not to prevent, but that the sums for each sex with the ratio considered of the living, females may die in lesser proportion than males. If I have wished to warn therefore, because I have seen some writers to produce an erroneous opinion concerning this thing.

Thus after I shall have treated our argument for the common hypothesis, I will reveal the manner by which generally and with total accuracy it is able to be accomplished.

§ 5. Let the number of all weddings have been from the beginning = n
 and precisely the number of all married together = $2n$;
 then let be supposed after a given run of years a part of them by
 death snatched and let the number of all surviving alive be = r
 and hence all of the deceased = $2n - r$;

³Essai d'une nouvelle analyse de la mortalité causée par la petite verole, et des avantages de l'inoculation pour la prévenir, *Hist. et Mém. de l'Acad. Royale des Sciences de Paris* 1760, pp. 1–45.

finally let the number of marriages yet until now subsisting be $= x$
 and precisely the number of all widows $= r - 2x$;
 Thus by these definitions, I say

$$x = \frac{rr - r}{4n - 2}.$$

I have given the solution of this question explained in the specimen *de usu algorithmi infinitesimalis in arte coniectandi* § 2. (see page 88 of this volume of the Journal) where indeed I have used with the usual limits to be employed according to the art of conjecture; but no one is to return the previous paper there who does not see. Further there will be the number of all widowed or

$$r - 2x = \frac{2nr - rr}{2n - 1},$$

of which the half likewise expresses the number either of widowers or of widows, since we set death to rage by scythe the same with regard to each sex.

§ 6. Thus therefore it is easy now to determine the number of marriages, which will have remained whole, for any remaining number of survivors whatsoever, and because that next number in order, to any year whatsoever of common lifetime, is discovered from the mortality table, the number of surviving marriages will be able to be indicated simultaneously for any year whatsoever. I shall choose the mortality tables by the most celebrated *Halley* constructed on behalf of the city of Breslau, which everywhere have been committed to type and exist by name in the distinguished work of *Deparcieux* to which the title is *Essay sur les probabilités de la durée de la vie humaine*,⁴ and I shall use them toward new tables composed paying attention to our mark. Certainly there is freedom to begin from any number of marriages whatsoever equally and as well from any common lifetime whatsoever. Consequently I shall put 500 first marriages entered upon among one thousand persons, of whom each single one must be 20 years old. But while in the table of *Halley* there are put 598 persons of 20 years, on the other hand I will suppose 1000, the individual numbers put by *Halley* will be increased in the ratio of the numbers 598 and 1000; I will disregard fractions and I will substitute a whole number for them, to which they are nearest or which observes more uniformity with respect to progression.

The table, which follows, is established out of four columns, each divided into two parts. The first column indicates age expressed in years; the second the number of persons surviving, the third the number of marriages subsisting and untouched by death; finally the fourth the number of widowed persons with no discrimination considered by sex, and it will be subdivided precisely into two equal classes, bereft of husband and bereft of wife. However in this count all are being reported, who have endured widowhood once, either they will have undertaken new nuptials or they will have retained the state of widowhood.

§ 7. The formula, which we have used, for the numbers of the third column by computation to be brought to light, namely

$$\frac{rr - r}{4n - 2},$$

adequately indicates these numbers not to be exact proportionals to the first number by an assumed choice, unless that beginning number be very great; but certainly then the last numbers will recede a very small amount from the identity of proportionality; to be noted further out of it, because we reject all fractions, very small errors greater than unity, yet smaller than two are able to fall for the numbers of the fourth column; those numbers generally are expressed by the formula

$$\frac{2nr - rr}{2n - 1} \quad (\S 5);$$

⁴A. *Deparcieux*, "Essay sur les Probabilités de la durée de la vie humaine. . ." Paris 1746. Supplements published in 1760.

therefore the number of widows is a maximum, when the original number of spouses has been reduced to half, that is, when $r = n$; but then the maximal number of widows becomes also, provided that of the living ones

$$= \frac{nn}{2n - 1}$$

or in our example $= 250\frac{1}{4}$, while yet the table itself will indicate the number 252. I remind this, lest any cause of abbreviation may assign very small admitted errors to the method itself.

I.	II.	III.	IV.		I.	II.	III.	IV.
Years old	Surviving lives	Remaining marriages	Surviving widows		Years old	Surviving lives	Remaining marriages	Surviving widows
20	1000	500	0		55	488	118	252
21	990	490	10		56	471	110	251
22	980	480	20		57	454	103	248
23	970	470	30		58	437	95	247
24	960	461	38		59	421	88	245
25	948	450	48		60	404	81	242
26	936	439	58		61	387	75	237
27	924	427	70		62	370	69	232
28	913	416	81		63	354	63	228
29	901	406	89		64	338	57	224
30	888	395	98		65	321	52	217
31	875	384	107		66	304	47	210
32	861	372	117		67	287	42	203
33	848	360	128		68	270	37	196
34	834	348	138		69	254	32	190
35	819	336	147		70	237	28	181
36	804	324	156		71	219	24	171
37	789	312	165		72	200	20	160
38	774	300	174		73	182	16	150
39	759	288	183		74	164	13	138
40	744	276	192		75	147	10	127
41	729	265	199		76	130	8	114
42	714	254	206		77	114	6	102
43	698	243	212		78	98	4	90
44	681	232	217		79	82	3	76
45	664	221	222		80	69	2	65
46	647	210	227		81	57	2	53
47	631	199	233		82	47	1	45
48	614	188	238		83	39	1	37
49	597	178	241		84	33	1	31
50	579	168	243		85	27	0	27
51	560	157	246		86	22	0	22
52	541	147	247		87	17	0	17
53	523	137	249		88	13	0	13
54	505	127	251		89	9	0	9

§ 8. The table, which we have given just now, begins from the age of 20 years: but it is not difficult to construct a new table for any other initial age whatsoever. Thus if the mind is to investigate the innate character of marriages for the age of 30 years, now the aforesaid table indicates the number of surviving marriages at this age to be 395; because indeed at the beginning the number of surviving living ones is always double the number of marriages, the second column should be specified to start from 790 betrothed; therefore the number 888 should be changed into the number 790, and afterwards all numbers set forth in the second column are reduced according to the same proportion; indeed all numbers of the third column must be held back; finally if the doubled numbers of the third column are subtracted from the reduced numbers of the second column in the aforementioned manner the numbers of the fourth column will appear. And thus easily tables will be constructed from the work for whatever common age of spouses.

§ 9. If now the time is sought, by which the first marriages, undertaken at the age of 20 years, their half parts must be destroyed by death, it will be clear by a single inspection of the table; because seeing that at the beginning 500 marriages are put, it presently ought to be seen in the third column, by what age 250 marriages may survive untouched by death:

I.	II.	
Years of age	Time for half of marriages to be destroyed.	
20	22 years	$4\frac{1}{2}$ months
25	19 —	7 —
30	17 —	2 —
35	15 —	0 —
40	12 —	11 —
45	11 —	0 —
50	9 —	6 —
55	8 —	8 —
60	7 —	4 —
65	5 —	6 —
70	3 —	8 —
75	2 —	6 —
80	2 —	0 —

and thus it will be evident it to happen between the 42nd and 43rd years of age or rather at the age of 42 years $4\frac{1}{2}$ months; thus therefore with an equal stake one will be able to contend a marriage, undertaken at the common age of 20 years, after 22 years $4\frac{1}{2}$ months yet subsists or not subsists.

Certainly the same question for any other age, only if it is the same for each married person, is solved in the same way. Let, for the sake of an example the age of each married be 40 years: our table indicates, 276 marriages to survive from this year of age, and this number to be reduced to half after twelve years elapsed as well as 11 months.

And thus I have constructed the adjacent table for any five year increment of age, where the first column of years indicates the age common to each married, however the second shows the time, expressed in years and months, within which probably the half part of the marriages must be destroyed. But by the method of interpolations quite accurately the work will be accomplished for any intermediate age.

§ 10. The aforementioned question concerning the time, in which the half part of the marriages is destroyed by undertaking from any sort of age whatsoever, is not to be confounded with our principal inquiry, which seeks the expected mean duration of marriages besides, whatever be the age of the spouses, but it is the same. Nevertheless it is easily foreseen the two questions to be able to differ between themselves not

very much; moreover the method for finding the mean duration of marriages is similar to that which we use toward the duration of expected life in addition; but this method itself requires, that previously any yearly destruction must become known and certainly this yearly destruction of marriages had been determined with no observations thus far yet, it should seem to me whether it is not able to be determined by calculation for whatever age, because the work has advanced out of a way of thinking; now therefore I shall indicate the manner, by which according to the calculation of the table, united with the sixth paragraph, the final mean duration of marriages may be sought for any age whatsoever.

§ 11. Let all numbers, defined in the third column of our table, be added by beginning from the given age of years all the way to the end and the sum of them is divided by the corresponding number of the given age: thus the *quotient* expresses, as it has been known, the mean duration demanded, if only single marriages, which are destroyed within one and any year, must be destroyed at once at the end of the year; but certainly since a nearly uniform destruction should have run through the whole of the year, this will be reckoned without sensible error to have happened in the middle of the year; hence it happens, that the aforesaid *quotient* is reduced by a half year or by six months. Thus therefore, if for the sake of an example the discussion is concerning the mean duration of marriages between spouses 55 years of age, the sum of the numbers of the third column should be taken from 118 inclusive to the end, which sum is 1208 and which now must be divided by the number 118 corresponding to the proposed age; the quotient will be nearly $10\frac{1}{4}$ expressing the equivalent number of years, from which, if finally the half year should be removed, $9\frac{3}{4}$ will remain or nine years together with nine months; therefore the mean duration of marriages between spouses of 55 years is nine years and as many months.

I.	II.	
Years of age	Mean duration of marriages	
20	23 years	10 months
25	21 —	3 —
30	18 —	10 —
35	16 —	8 —
40	14 —	9 —
45	12 —	10 —
50	11 —	1 —
55	9 —	9 —
60	8 —	1 —
65	6 —	2 —
70	4 —	6 —
75	3 —	4 —
80	3 —	0 —

Thus therefore in the manner of the example produced, for any age again by a five year increment, I have formed the adjacent table, of which the first column of years indicates the age of each spouse, the second certainly shows the mean duration of subsisting marriages with those diverse ages; interpolations will complete what is remaining in this work.

§ 12. Now in order that it may be apparent in a certain way, until where this theory of ours may be consistent with observations, I will lay out an example, which I read recently in regard to transactions in Bern, where the enumerated surviving marriages is said to be 1053 in Lausanne, and there 49 marriages to be consecrated annually; thence it follows the mean duration of these marriages recently undertaken to be

approximately 21 years together with six months;⁵ this mean duration is a slightly more than that which is fitting to the age of 25 years. But certainly it is not at all unlike the mean age of all marriages, which are consecrated, to be nearly of 25 years, so that our calculations with the observations themselves may not fit badly. Now certainly there is no leisure to investigate other examples in the writings of diverse Authors. If any others wish to undertake that of another business by themselves, it will be easy to form one's own opinion to this, only they must remember the mortality to be slightly different in diverse regions and also the duration of marriages to increase or be diminished according to the degree of mortality.

§ 13. Now if further we desire the sum of continuing marriages in a city, which provides 500 new marriages yearly, but at the same time it must be supposed no other marriages to be celebrated than among persons 20 years old, nothing other must be done, than that we multiply the mean duration of the new marriages for the year by the number of these marriages, that is, $23\frac{5}{6}$ by 500; the product 11917 gives the sought sum: this will be the sum of all continuing marriages: but the sum of all persons joined by marriage contract will be 23834. If indeed another common age is put to every new betrothed, the table of the eleventh paragraph will serve the interests to the same question; for everywhere the number of annual marriages will be to the sum of all the continuing marriages as the time of one year to the time of mean duration which has been recorded in the second column of the table. Let us assume therefore individual marriages to be postponed to the end of the thirtieth year, now it will be proper to multiply the number of annual marriages by $18\frac{5}{6}$. But the number of annual marriages now itself will be less by the ratio of the surviving alive, that is, by ratio of the number 1000 to 888, which correspond to ages of 20 and 30 years; therefore the number of annual marriages now will be $\frac{888}{1000} \times 500$ or 444, which multiplied by $18\frac{5}{6}$ gives 8362, when before it will have been 11917; furthermore can it be that marriages of the first class are able to be presumed much more fruitful, than marriages undertaken finally with the thirtieth year of age ended? If some city will have received 2300 children each year from 500 annual marriages, with each contracted before the end of the twentieth year of age, this same city will gain perhaps scarcely beyond 1000 children every year, if each and every marriage should be undertaken with the thirtieth year of age elapsed.⁶

§ 14. I shall add a word concerning the number of all widows of each sex, which will pertain to the same city of which we have spoken, if they are assumed to preserve widowhood perpetually, or rather if all persons are classified in widowhood one time of the men or women permitted to this census. But if however in the table of the sixth paragraph the numbers of the fourth column are added; the number 10326 will arise which will express very nearly the number of all widowed persons to the end of age 89 years; to this just about 24 surviving ones that transgressed the age are able to be added, so that the sum of all widowed persons in the city should be able to be reckoned 10350. Therefore all persons of a character joined by a marriage contract will have ratio to all widows just about as 23834 to 10350 or nearly as 23 to 10. Therefore these will keep themselves, if marriages individually are assumed to be celebrated from the age of 20 years for each sex. Another age may have another table and other numbers, which however are obtained by a trivial change; I may be excessive, if I should mention everything briefly, for as many questions as possible remain, which are determined by one calculation and even better than by observations no matter how regular.

§ 15. I proceed to the other kind of marriages undertaken between spouses of different age. Ordinary marriages are of this kind, seeing that generally the husband surpasses the wife in age. But the risk of death has been known to increase with age, besides which, all things being equal, even more risk is to men than to women. Thus therefore if greater age is assumed to the husband, these will depart more easily and

⁵*Translator's note:* Since $1053/49 = 21.49$, Bernoulli is assuming a stationary population.

⁶*Translator's note:* Bernoulli does not explain the source of these counts of children. The number of children born should be proportional to the number of intact marriages of women of childbearing age. It is the case that the ratio of the number of marriages where the spouses are between the ages of 30 and 39 inclusive to the number of marriages where the spouses are between the ages of 20 to 39 is approximately 23 to 10.

more quickly from life and by far they will leave behind more widows behind themselves, than they should become widowed. I feared for my part, lest that inequality between widowers and widows inflict too much difficulty in our argument; but the nature of the thing itself has assisted with it, because the differential equation among three indeterminants admits not only a separation but also an integration, as if by good fortune. I have had similar fortune in a specimen, the title to which is, *Essay d'une nouvelle analyse de la mortalité causée par la petite verole etc.* Mem. Acad. R. Sci. Paris inserted in the year 1760, of which now I have made mention above § 3. Now certainly it is understood, by my method of the infinitesimal algorithm having been used, which requires, that a very great or as if infinite initial number of marriages must be admitted. That method, with names changed only, I have related entire in the preliminary specimen *de usu algorithmi infinitesimalis in arte coniectandi* from the seventh paragraph to the end (see again pag. 87 in this volume of commentaries). Here are husbands and wives, whereas there black and white tickets; here marriages, whereas there pairs of associated tickets; here finally greater mortality of one or the other sex, whereas there greater facility for the extraction from the urn or more propensity in egression.

§ 16. Now let the number of initial marriages be again assumed $= n$
and let these marriages individually be so constituted, that all husbands at least are of the same age among themselves together as well as the wives, but yet the age has been separated in one sex and in the other. Then, with a given elapsed number of years,
let the number of surviving men $= s$
and the number of surviving women $= t$;
moreover the two numbers are distinguished as a result of the mortality table: next it is sought how many marriages among all these survivors will be entire or consequently always untouched by death, further how many widowers or at least widowers once and how many widows or at least widowed once. Let again the number of surviving marriages be assumed $= x$;
thus the number of surviving widowers will be $= s - x$
and the number of surviving widows $= t - x$.
Thus with these established we have demonstrated at the end of the seventh paragraph of our preliminary specimen *de usu algorithmi infinitesimalis in arte coniectandi*, to be

$$x = \frac{st}{n}.$$

Consequently all becomes known with proven or given numbers s and t .

Hence it is clear, not to be necessary, that the ratio of a different mortality, which arises from the difference of age, must be had directly with regard to the opposite sex; yet no one does not consider this fact to have been from it implicitly, because a relationship must be supposed between the numbers s and t , which necessarily involves different deaths: in fact our formula, however much the most simple, thus is general, that likewise in addition the ratio is able to be had of another different mortality, which reckons origin from a difference of sex, accordingly the mortality tables should not disappoint, in which men are distinguished from women. That in fact again we are able to use the tables of *Halley*, we shall consider only that one difference of mortality, which is owed to the difference of age, because we have done the same when we have treated concerning marriages between spouses of common age; thus by so much better the different kinds of marriages will be able to be compared among themselves.

§ 17. It remains that I should show the method, by which another table, similar to that which I have joined with the sixth paragraph, is able to be constructed for an unequal age of spouses: indeed any difference whatsoever of either age requires a new calculation; yet it will be sufficient to go by periods of five years, because interpolations will exhibit the intermediate states accurately enough. An example, which I shall produce now, will be the form of all.

I. Age in years		II. Surviving		III. Remaining Marriages	IV. Widowers	V. Widows	I. Age in years		II. Surviving		III. Remaining Marriages	IV. Widowers	V. Widows
Men	Women	Men	Women				Men	Women	Men	Women			
40	20	500	500	500	0	0	70	50	159	289	92	67	197
41	21	490	495	485	5	10	71	51	147	280	82	65	198
42	22	479	490	470	9	20	72	52	135	270	72	63	198
43	23	468	485	455	13	30	73	53	123	261	64	59	197
44	24	457	480	439	18	41	74	54	111	252	55	56	197
45	25	446	474	423	23	51	75	55	99	244	48	51	196
46	26	435	468	408	27	60	76	56	88	235	41	47	194
47	27	424	462	393	31	69	77	57	77	227	35	42	192
48	28	413	456	377	36	79	78	58	66	218	29	37	189
49	29	401	450	361	40	89	79	59	56	210	24	32	186
50	30	389	444	345	44	99	80	60	46	202	10	27	183
51	31	377	437	330	47	107	81	61	38	193	15	23	178
52	32	365	430	314	51	116	82	62	31	185	11	20	174
53	33	353	424	299	54	125	83	63	26	177	9	17	168
54	34	341	417	283	58	134	84	64	22	169	7	15	162
55	35	328	409	268	60	141	85	65	16	160	5	11	155
56	36	316	402	254	62	148	86	66	12	152	4	8	148
57	37	304	394	241	63	153	87	67	9	143	3	6	140
58	38	293	387	228	65	159	88	68	6	135	2	4	133
59	39	282	379	215	67	164	89	69	3	127	1	2	126
60	40	271	372	202	69	170	90	70	2	118	0	2	118
61	41	260	364	190	70	174	91	71	2	109	0	2	109
62	42	249	357	178	71	179	92	72	1	100	0	1	100
63	43	238	349	166	72	183	93	73	1	91	0	1	91
64	44	227	340	154	73	186	94	74	0	82	0	0	82
65	45	216	332	143	73	189	95	75	0	74	0	0	74
66	46	205	323	132	73	191	96	76	0	66	0	0	66
67	47	194	315	122	72	193	97	77	0	58	0	0	58
68	48	183	307	112	71	195	98	78	0	48	0	0	48
69	49	171	298	102	69	196	99	79	0	41	0	0	41

Let us assume new marriages to be undertaken between five hundred young women each twenty years of age and just as many men of forty years. Thus again there will be $n = 500$; then indeed the numbers s and t come to be sought after several years elapsed or so much the numbers of men surviving as of women; each number is discovered by a very easy calculation with the help of the second column of our table attached to our sixth paragraph. As a matter of fact the number t is obtained assuming half of any number expressed in the aforementioned second column; if then that half number is multiplied by $1000/744$, the number s will be had; but that multiplication should be made therefore, because the original number of men is supposed = 500; and thus the second column will be formed for the men as well as the women surviving for any year whatsoever. With this completed, the third column will be formed, which indicates the remaining marriages, by assuming the quantity $\frac{st}{n}$ for any year. Finally the fourth and also the fifth columns, which indicate the number of widowers and widows, are constructed by subtracting the number of remaining marriages from the number of men or from the number of women surviving. But the ultimate numbers, to which the mortality table of Halley does not extend, although by themselves they may be hardly ever of importance, yet I have considered to be joined and to be built upon from other mortality tables.

§ 18. The use of this latter table is the same as of the preceding: therefore whatsoever we have advised from the seventh paragraph to the end of the fourteenth paragraph, it furthermore, mutatis mutandis, will be able to be determined for existing kind of marriages. Thus, for the sake of an example, when I form the question in the ninth paragraph, concerning the designated time, where the half part of marriages, between persons 20 years old, must die, I have discovered that time of 22 years together with 4 and a half months; in the existing kind of marriages, which happen between wives 20 years old and husbands of 40 years, that time, which appears out of a single inspection of the third column in the last table, is contracted to 16 years together with four months. If a similar question is made of marriages between women of 40 years and of husbands of 60 years, the same third column indicates then 202 marriages to survive, and it to be reduced to 101 by the time of 9 years together with one month. And thus with no work a table will be constructed similar to that which I have set in paragraph nine above, since it suffices to have instructed. But since the discussion is principally concerning the mean duration of marriages joined for a different age, I shall not decline that too little work.

§ 19. This question is completed, when husbands surpass wives by an age of 20 years, clearly by the same manner which we have used § 11, where an equal age for each spouse was assumed. Thus for the initial age of 40 years in men and of 20 years in women, the sum should be received of the third column in the preceding table, which becomes = 9207 and this divided by the number of initial marriages or by 500; the quotient will be $18\frac{41}{100}$ years or approximately 18 years together with 5 months; 6 months removed from here, 17 years together with 11 months remain for the mean duration sought.

Years of age		Mean duration of marriages	
Men	Women		
40	20	17 years	11 months
45	25	15—	8—
50	30	13—	9—
55	35	11—	11—
60	40	10—	0—
65	45	8—	1—
70	50	6—	2—
75	55	4—	10—
80	60	3—	6—
85	65	2—	6—

And thus I have constructed a very small adjacent table, which now it is permitted to compare with the similar table adjoined to the eleventh paragraph, so thus the distinction is evident, which intercedes between each kind of marriages, by reason of the duration to be expected of them for whatever time beyond. By no means small is that distinction, and more it is going to be, if likewise the ratio had been considered of increased mortality with regard to men and the mortality tables had been used precisely, in which each sex is distinguished in turn and the number of surviving ones from the men and from the women is indicated separately.

§ 20. It is pleasing finally to establish for a select few a comparison between each kind of marriages; the first for which two spouses are of the same age, the other for which the husband is assumed older than the wife by twenty years; and thus concerning each mean position they, who dread new calculations, will be able to bring judgment without great error.

a) Now we have talked concerning the different mean duration for both kinds of marriages. Thus for the sake of an example, a wife of 55 years married to a man of the same age not without reason promises to herself a longer duration of her own marriage to another by so much, than if she was married to a man of 75 years.

b) If we imagine two cities, of which the one supplies yearly 500 marriages of the first kind, the other just as many marriages of the second kind, the former city will enumerate 11917 marriages, the other none except 8958 (§ 13.) if only each city is assumed to remain in its state. But we speak concerning the unions undertaken by each spouse of the first reciprocally, which certainly constitute the greatest number.

c) Again one and the same city, which yearly is able to furnish 500 marriages of the first kind, will not be able to supply beyond 372 unions of the second kind, because namely of 500 adolescents of 20 years, none except 372 will attain the age of 40 years. Thus a fourth part of the young women or bachelors is forced to preserve or to be married to the widowed. Hence the ratio, because many more widowers than widowed may undertake marriages again, even if the number of the former is less by far than the number of the latter.

d) Now we speak also concerning the widowed persons of each sex; among these we report all those, who have suffered widowhood once, whether they should preserve that or they should have recourse to a second marriage. But we have seen § 14 that for 500 yearly marriages of the first kind, the sum of all persons is, for which we have said only, of the widowed = 10350, or of 5175 widowers and as many widows; but the thing by itself holds different by far if the discussion is concerning marriages of the second kind. As a matter of fact the sum of the widowers then is less by far than the sum of the widows, seeing that our table § 17 indicates only 2154 widowers; but the sum of all numbers of the fifth column is = 7949 and that number of widows yet should be increased, on account of the surviving widows after the end of the column, I count 197, by which fact the number of all widows = 8146. Thus certainly in both cases the sum of all widowed persons is just about the same; but if we compare the widows with the widowed men, we see in the latter case the number of widows to be nearly the quadruple of the number of widowed men, while in the first case they are equals between themselves. Therefore in this manner in a city, where 500 marriages of the first kind annually may be celebrated, there should be 11917 husbands, just as many wives, 5175 widowers and just as many widows and the sum of all 34184; indeed if there are 500 annual marriages of the second kind, then there should be 9207 husbands, just as many wives, 2154 widowers and 8146 widows and the sum of all 28714.

e) In a city, which is assumed at a continuing state, if every year 500 new marriages of the first kind are celebrated, it is necessary that just as many marriages yearly must be dissolved; 250 husbands will die, who will leave behind just as many widows after themselves and also 250 wives, who make equally just as many widowers. Indeed if 500 annual marriages of the second kind will have existed, a death yearly will remove just about 345 husbands, 155 wives, 155 widowers and 345 widows:
for because the number of surviving men = s,

the number of surviving women = t
 and the number of remaining marriages (§ 16.) = $\frac{st}{n}$
 the decrease of marriages will be, as the number of the first to the number of the latter, that is, as = s to $\frac{st}{n}$
 or as = n to t ;
 this decrease of the husbands should be expressed by

$$\frac{-t ds}{n};$$

but the decrease of the wives will be

$$\frac{-s dt}{n},$$

whence the entire number of deceased husbands

$$= - \int \frac{t ds}{n}$$

and the entire number of deceased wives

$$= - \int \frac{s dt}{n};$$

because certainly the general relation between s and t is not given, except by the numbers of the mortality table, the aforesaid integrations are not able to be made in any other way than by parts, and thus I have discovered numbers which only I have exhibited.

Therefore there appears out of any specimen without exception into that thing, to be many variations and vicissitudes in regard to human kind, which should be able to be determined more accurately and more usefully by a single computation, than it has been able to be made by innumerable observations thus far.