

Absent*

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Encyclopédie Méthodique
Mathématiques. Tome I, 1^{ère} Partie, 1783

ABSENT (*Calculus of probabilities*)¹

We have believed to have to add to this article, drawn from the first edition of the Encyclopedia, the following reflections:

I. The new Theory of M. de Buffon has proposed, does not appear to be able to be admitted.

1° It is inexact in itself, since it tends to confound two things of different nature, probability & certitude.

2° It is able to serve only to simplify the calculation, in the case where the result, to which this hypothesis would lead, would differ only in an insensible manner from the rigorous results: thus, it must not be employed to resolve any of the difficulties which are able to raised on the principles even of the calculus.

3° One is permitted, in the calculation, to employ as absolute an approximate determination, only for a quantity of which one knew a value slightly different from the real, & of which the real value is uncertain; now here the value of certitude is determined & equal to unity. Thus, there is no reason to suppose in the calculation, either any probability equal to unity, or certitude equal to a fraction little different from unity.

4° When one is permitted to neglect a quantity, it is always for the reason that one regards it as null, with respect to that which one wishes to know, & it is that which in not able to take place here. Every time that the probability of an event is little different from unity, the probability of the contrary event is a very small quantity. Thus, I am well able to regard as being equally probable the events of which the probabilities $\frac{10^{1000}-1}{10^{10000}}$ & $\frac{10^{10001}-1}{10^{10001}}$ are slightly different.

But it is not likewise with the contrary events, of which the probabilities $\frac{1}{10^{10000}}$ & $\frac{1}{10^{10001}}$ are in the ratio of one to ten.

5° A *maximum* of probability is an expression which is not able to be understood in mathematics; the *maximum* of the probability will be 1, & it is not able to be attained. But one can fix a *minimum* of probability, that is to say, a probability below which, for example, it can be permitted neither to condemn

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¹*Translator's note:* The article ABSENT consists of three parts. The first is that by d'Alembert written for the *Encyclopedia* of Diderot. The second is contributed by Diderot. Finally, the third parts is due to Condorcet.

a man, nor to strip him of his wealth, nor to repute him dead. This idea is absolutely opposed to that which Mr. Buffon proposes, & it is the only one which one must admit.

II. In the questions relative to the wealth of the absentees, it is necessary to examine separately the following three hypotheses:

That where the *absentee* would return at the end of a certain time, that where the *absentee* will never return, but where the period of his death is known, & that where one ignores this period in perpetuity.

This put, we will demand first according to what principle one must regulate the administration of the wealth of the *absentee*, in order that, in the case where he would return, he had experienced no injustice.

It is clear that on his return, he would take back his claims on all his wealth, & that the question is therefore only to determine at what period one ceases to leave them sequestered, & to permit his heirs to divide them, & to dispose of them, in giving, for their surety, the proper deposits, & at the end of which time one can even excuse them formalities.

For the 1st case, one sees that, in the moment where the life of a man is uncertain, one does an injustice to him, in the case where he would live, in permitting his heirs to dispose of his wealth, & that, if he is dead, one does one to his heirs in depriving them of the succession. Let therefore a be the probability of risk to which one will expose an *absentee*, in permitting to distribute his wealth b at the end of a number n of years of absence; the value of the loss to which he would be exposed will be expressed by ab , & if u is the probability that he is living, his risk will be expressed by uab .

Now, what is the loss of the heir, if one refuses to put it in possession? The wealth is here always b : let $\bar{1} - r$ be, the probability that he will always enjoy it, without being obliged to render it, that is to say, the probability that the *absentee* will not return; $1 - r \cdot b$ is the wrong done to the heir.

But this wrong is not irreparable, since, if one renders b to him, increased by interest, the year after he will have nothing lost; let therefore u' be the probability that he will not die in the year; $\bar{1} - u' \cdot \bar{1} - r \cdot b$, will express the risk that one makes him incur, by not remitting to him the disposition at the beginning of this same year; thus, in order to deliver the wealth of the *absentee* to the same heirs, by requiring a deposit, it is necessary that $\bar{1} - r \cdot \bar{1} - u' > ua$. The case where one can cease to require the formalities of deposits, will be resolved likewise, with that exception that a must be then greater. One can, without much error, suppose $1 - r = 1 - u + \frac{1}{2}u$: in fact $1 - r$ is the probability that the *absentee* will never return; but the probability that he is dead is $1 - u$, & one can suppose that, if he is living, there will be as great odds that he will return, as that he will not return.

This 1st condition is not sufficient; it is yet necessary that, in the case where one would know one day the fixed period of the death of the *absentee*, there resulted the dispositions made of his wealth, no injustice in regard to his heirs.

It is clear that the wealth must be divided definitely, as if the succession were opened on the day of his death. Thus, suppose that b is, in a given period, the portion of the wealth which heir A demands, & that, by the death of A, this

portion would pass to B , the wrong done to A is here as above $\overline{1-r} \cdot \overline{1-u'} \cdot b$. In order to evaluate the wrong done to B , one will find:

1° That a can express the danger to which one exposes him;

2° That this danger takes place only as far as A would die before the *absentee*; therefore, if u'' expresses the probability that A dies before the *absentee*, one will have $u''ab$ for the risk to which B is exposed, if one gives a part of the inheritance to A . Thus, it will be necessary, in order to leave to A the disposition of his portion, that $\overline{1-r} \cdot \overline{1-u'} > u''a$.

This condition must be fulfilled in all the combinations which can take place in the order of mortality of the different heirs.

Suppose finally that there is a question to regulate the division of the wealth, by having regard equally to justice under the assumption that the *absentee* never returns, & that one must be ignorant of the period of his death. We know, 1° the period where one can, without injustice, make the division of his wealth. Let b be the wealth; since there is $1-u$ to wager that he is dead, it will be necessary first to take a part $\overline{1-ub}$.

2° If, from the uncertainty of his lot until this period, there is for one or two other periods $A \cdot B$ some variations in the order of the succession, it will be necessary, if $1-u \cdot x$, $1-u \cdot \overline{x+z}$ represent the probability of the death in these periods, to divide $\overline{1-u} \cdot xb$, as in the period A ; $\overline{1-uz} \cdot b$, as in the period B ; $1-u \cdot 1-z-x \cdot b$, as in the period of the distribution. The rest can be distributed provisionally, but always on the condition that, if a change in the order of succession occurs, a part of this remainder proportional to the probability that death has not arrived before this change, will be distributed as it would have been, if one had learned that his death is arrived after this change. Finally, when the probability of the death of the *absentee* will have surpassed this *minimum* of moral certitude of which we have spoken above, it will be necessary to distribute definitely all the wealth, following the same rule²: whence there results that it is the only period in which the formalities of deposits, &c. must be eliminated.

These principles suffice to resolve all the questions which can present themselves on this object; they limit themselves to this very simple rule, to suppose proportional to the probability of life & of death of the *absentee*, the wrong which results for him or for his heir, on the assumption that he is dead or living in each period that one considers, & to await, for each irrevocable decision, that the probability of death be beyond the *minimum* of probability, for which it can be permitted to deprive a man of his claims. (*See the article* PROBABILITÉ.) The questions relative to the distribution of the successions which can fall due to an *absentee*, to the claims of his creditors on his wealth, &c. must be resolved by the same principles.

We ourselves are limited here to some general principles, because their application in practice, depends on the laws of each country respecting the deposits, on the prescription, on the order of succession, on the more or less of freedom

²See, respecting the general principle to distribute the sums proportionally to the probability of the claim, *the article* PROBABILITÉ.

of testamentary dispositions. Thus, in each nation, in order to make, respecting this subject, a law conform to reason & to justice, it would be necessary, after having resolved, in all cases, the questions which can present themselves, to seek to deduce from all the results which one would have obtained by the calculus, some general & simple rules, which could render liable to commit some injustices, only in some cases nearly impossible to suppose. (*M.D.C.*)