LES RÉCRÉATIONS SCIENTIFIQUES

GASTON TISSANDIER
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We arrive to the *game of the needle*. It concerns a true mathematical recreation, of which the result, indicated by theory, is well known to produce astonishment.

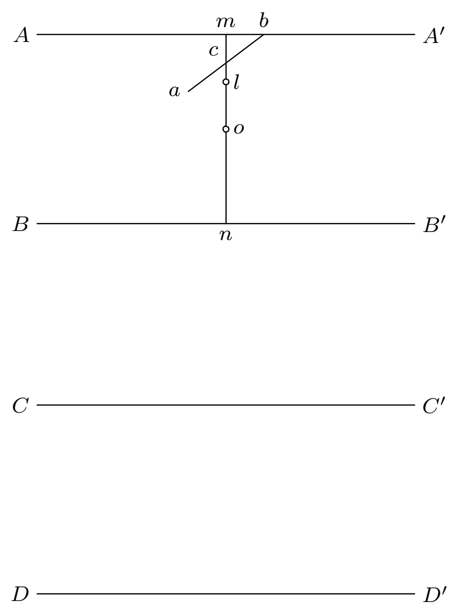

FIGURE 1. Disposition of the outline destined to serve in the game of the needle, relative to the calculus of probabilities.

The game of the needle is an application of the diverse principles that we have posed on probabilities.

If one traces on a sheet of paper a series of lines AA', BB', CC', DD' equidistant parallels, and if one projects at random onto this sheet of paper a needle ab entirely cylindrical of which the length will be equal to the mean of the distance of the parallels, one will find this curious result.

If the experience is prolonged a rather long time, in order to fix the ideas, if one projects the needle one hundred times at random, it will happen that in these one hundred tests the needle will encounter any one of the parallels a certain number of times. By dividing the

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number of tests by the number of encounters, one will obtain as quotient a number which will approach so much more the value of the ratio of the circumference to the diameter, as one will have multiplied more the tests.

This ratio, according to the principles of geometry, is a fixed number of which the numerical value is: 3.1415926.

After one hundred tests, one finds generally the exact value to the first two digits: 3.1.

How to explain this unexpected consequence?

The application of the calculus of probabilities gives the reason. The indicated ratio of the encounters to the number of tests is the probability of this encounter. The calculus seeks to evaluate this probability by making the enumeration of the possible cases and of the favorable events.

The enumeration of the possible cases requires the application of the principle of composite probabilities. One sees easily that it suffices to consider the chances that has the needle to fall between two determined parallels AA' and BB', next that it suffices likewise to consider that which is passed in the interval mn equal to the equidistance. For an encounter, it is necessary therefore:

1° That the middle of the needle falls between$m$ and l middle of mo;

2° That the angle of the needle with mo is smaller than the angle meb.

The evaluation of each of these probabilities and their combination by multiplication, according to the principle of composite probabilities, give finally for expression of the probability the number π.

This curious example justifies the theorem of Bernoulli relative to the multiplication of events: there is no limit to the approximation of the result when one prolongs rather far the test.

When the length of the needle is not exactly the half of the distance of the parallels (it is able to be any, provided that it remains inferior to this distance), the practical rule of the game is the following:

It is necessary to multiply the ratio of the number of projections to the number of encounters by the double of the ratio of the length of the needle to the interval of the parallels. In the particular case above, the double of the last ratio has for value unity. We will give a numeric application cited by the authors.

With a needle of 50 millimeters of length projected 10,000 times, in a series of parallels of which the distance was 63.6^{mm}, one has found a number of encounters equal to 5,009.

One take the ratio $\frac{1000}{5009}$, one multiplies it by the ratio $\frac{1000}{636}$ and the product is: 3.1421.

The true value is: 3.1415.

One has an approximation of $\frac{6}{10000}$.

The dimensions indicated in this experience are those which present, for a determined number of tests, the more chances to obtain the greatest approximation possible.