

DOUTES

sur les bases du calcul des probabilités*

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All that which is not demonstrated, is either possible, or probable. The possible being all that of which the contrary implies no contradiction, & the contrary of the probable remaining possible, it follows that there is this conformity between the very pure possibility & probability, that all that which is possible is not at all probable, but that all that which is probable has been possible before the probability occurred; the one is therefore to the other as species is to genus. There remains to see what is the difference that one can establish between the one & the other; because under this completely general aspect, probability, the same is only a sort of possibility, & it would be true to say, that between two enunciated contradictories necessary or demonstrated, the space which separates them is filled only by some simple possibilities.

There is nothing in the world which can not & must not be resolved in representations, & in combination with representations. If each representation or combination of representations is not a fact or an experience (witness the purely speculative truths, the arbitrary notions, axioms) each fact or experience, in order to be some thing with respect to us or to any intelligence whatever, must be converted into judgment or into proposition. To say that a fact past, present or to come is probable, is to say that a proposition is probable. We affirm or we deny the existence of it by some considerations which render it to us probable or improbable. These considerations can never be but the analysis of the subject, of some manner which we managed to make it. Two things influence therefore on the relative truth with respect to us, of a proposition; the number of characters of a subject which enter as predicate, & the surety more or less great of the method that we have employed in order to find these characters. This method, as one knows, is either indirect or direct; indirect when one shows that the contrary of a proposition that one could refuse to admit is absurd or contradictory; direct when one is assured of the characters of the subject either intuitively or discursively, that is to say by the reduction of the individual to the genus or to the specie. As for the number of characters of which one arrives to exhaust the subject, it can be more or less great with respect to the totality of those which it contains; as for the method, one can

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have some doubts either on the legitimacy of the consequences which one draws *ab absurdo*, or on the analysis which one makes on the subject, or finally on the principles which one poses in order to supply by the syllogistic artifice to intuitive analysis, & on the consequences which one draws from it.

Suppose your subject perfectly equal to the sum of your predicates, & suppose that the method by which you yourself are served in order to analyze it has been deficient at no place, your proposition will be demonstrated or necessary, & such that that which is contradictory to it will be impossible. Suppose that there is in the subject of which your proposition expresses the nature or the action, an unknown side more or less considerable & which can, unknown to you, derange your conclusions, & suppose that there is in your manner to proceed in order to establish the known side of the subject more or less defects, not established (because if they were so, either you would remediate, or you would abandon entirely the method which is subject) but possibles or probables; in each case your proposition is only probable, of that probability that I will call objective, because on one side the subject is counted to contain much more than that which one affirms of it, is beyond us, it must have its consistency so to speak, its nature, its capacity determined without us, & independently of that which we can think or say of it; on the other, it must have in logic or in the reunion of all the laws of reasoning a perfection so to speak absolute, distinct from the sentiment that we have of it, & by which we ourselves perceive it. When a person would be in a state to say that which it was necessary in a proposition in order to be demonstrated, & to a logic in order to contain the secret of the truth, one imagines that this degree of perfection must exist somewhere, that it must have necessarily a point where all that which belongs to the demonstration is found concentrated. Annihilate all the finite & limited minds or all the subordinate capacities to appreciate the highest degree of perfection in the art of reason & of characterizing the truth, but conserve the idea of a supreme intelligence; since this moment this period where all that which is necessary in order to form the truth, is reassembled, is found, & exists. First of all the sciences, that of reasoning, has its absolute perfection that one could call its infallibility; consequently each probability which will be only a part of this science, will be an absolute objective probability, which has a sort of existence beyond us & in the nature even of things. I believe therefore (because that which I just said suggests it) that there is beyond that a subjective or relative probability, & I intend thence an estimation which we make on the value of a proposition, not following the place which it occupies among all the degrees of evidence possible, that which would render it objective, but according to some principles founded on our manner to be & to think. It would be useless that the truth or that which approaches it more or less, existed, if the intelligence which must gather the fruit of it was not warned in a manner or other that it possesses it. As the *conscium* of ourselves is entirely different from our existence, & however it would leave this existence either useless or inexplicable, if it would not be found in us, there is likewise the *conscium* of the truth or of the error, which without being the truth or the error itself, is so inseparable from it that without this thoughtful view of one & the other, it would be impossible to attribute to them some existence & some reality, or to say at least in what it would consist. As much one has done well since Descartes to reject as character distinct & useful from the truth, evidence, as it is necessary to acknowledge, that after being convinced to have found the truth by leading reason itself from the path

that one has taken to arrive there, one can not abstain from experiencing this sentiment of the evidence which is very different from the causes which must produce it. All the degrees which from complete evidence, make us pass to the last & to the most feeble of its nuances, measure the vast field of subjective probability which is composed of all the ways of which the true, according to that we believe more or less to have it, can affect us.

One would believe therefore that a sentiment which without being the definition of the true, is as the reflection of it in our soul, would always be regulated exactly out of that which we know of the true, to find its explication & its measure. It is however that which does not always arrive; thence comes that the true can often not be likely, & the likely takes the form of the true, & to usurp the power; thence comes that this same portion of the objective probability which is manifest to us, is often in opposition with the probability which combats it, & goes much beyond of that which the other would permit it to be. By a sequence of this peculiarity attached to our judgments, not only the limits of the true & of the false, of the demonstrated or of that which it is not, are unfortunately contested on diverse subjects, & have no invariable basis, but in order to confine me in my object, all the degrees of probability are vague, & the same proposition will traverse them all according to what you will present to different persons, or to the same person placed under all the circumstances which imagination can diversify to its pleasure.

It would be very useful to study this moral mechanism more thoroughly than one has done to here, & to seek to know all those principals of assent to some probable propositions, hidden in our heart, & independent or very nearly, it is necessary, from all that which these propositions can be worth in themselves. From these principles some are universal & based on the nature of man; the others particulars & shared among the diverse species or classes of men who form society; of others finally individual & resulting from the difference which there is from one man to another. One senses that I outline here the vast matter of prejudices; because what is a prejudice if not a judgment of which the reason is, either in whole or in part, subjective?

There are in all men certain instincts or tendencies which at the moment where a proposition interests them, imprint in him a character of probability which it would be impossible to restore to some distinct ideas, or which at least surpasses by much the degree of probability which it would conserve again after analysis. In being born we are again only in principle all that which we must be. This which in the following & in all the duration of our existence will become distinct, intelligible, explicable, is first & can be only confused perception. It is in this enveloped & automatic state which exists within us & understanding, & sensibility, & morality or conscience. It is in this same apparent disorder that the exterior objects and the impression which give birth to it, are addressed in these three faculties of the soul; each of them attract (if I dare to express myself thus) to it by a secret & rapidly felt analogy, all that which is of its causes; the propositions without number which are formed from all the possible combinations of these objects & of these impressions, go to arrange themselves under these three sources that a secret analogy indicates to them; they have another character than the one that gives them this association or sightless subordination which is made immediately without us in infancy, & which more or less during all our life restrains us & dominates us. There is likewise none of these propositions which in certain regards

belong to these three united powers of the soul, & which consequently appear to us more or less probable according to all the diversity of the primitive dispositions which modify it. It will suffice to indicate these dispositions in order to make understood how they must without us & without our knowledge determine the acquiescence which we give to certain propositions.

Is the question to understand it? Often he will believe, because destined to find the truth, loving & burning with the desire to possess it, he is persuaded much more easily to have met it as he wishes it more ardently, which he joins to that illusion which is dear to him a sort of laziness & of fear of the work or of the reverse attached to the examination; he will believe, because he senses that his natural state is less to doubt than to believe, in a word because he contains a crowd of tendencies of this order which all strangers as they are to such given proposition, influence much on the place which he assigns to it. One combats these blind principles of assent or of doubt as much as it is possible when one is a philosopher, & in measure as one is more; but each of us, if he reflected on himself, senses well that he does not always succeed to shake this authority so ancient & so hidden that they had on him.

Is the question of sensibility? it is chiefly the great thermometer of probability & of the facility more or less great with which all sorts of assertions go to place themselves on the ladder of truth. A person is not without passions, without interests, without hope or without fear, without a constant disposition in each return to faith, to his repose, to his glory, to his pleasures; there is no proposition at all so indifferent from the first glance, which in the prodigious diversity of the fibers of sensibility does not leave to strike someone, & this would be a common place too hackneyed than the one where I would fall, if I would stop myself a moment to prove that one believes nearly always that which one wishes, that which one hopes, & that in order to be very nearly sure of the opinions or of the maxims which will find access near to some men, it would suffice nearly always to know what interest they can have in adopting them.

Is the question finally of conscience, as a third field of operations for the soul? it also, according as it is more or less illuminated or delicate, gives to such proposition out of its simple enunciation & without ulterior analysis, a degree of probability which it would have never had without the point of contact where it is found with this principle of the moral activity of the soul.

Such are the secret causes which act on all men, & create for them that which I have call a subjective probability; it is of particulars which are taken on the entire classes of men exclusively from the others. Each age, each state or condition, each situation of life has, so to speak, its probabilities which are not for others of them. That which appeared probable to us at fifteen or twenty years, no longer appeared to us at fifty or sixty; time & its gentle wing have dissipated the soft illusions & the agreeable dreams of irresponsibility, of inexperience & of that heat of the blood & of the head which exhilarated & realized all. Why the great & the small, the powerful & the weak, the rich & the poor, the noble & the one who the heavens made be born in obscurity, have they a way to see & to think so different & with interests so opposed? It is that there is a logic so to speak of ranks & of conditions, & in order to be in the state or in the church, man of the robe or of sword, negotiator or man of letters, it is otherwise to contract the engagement, at least to incur the risk & to be exposed to the continually testing to see all his opinions to take the hue & the color of the most strange circumstances to

the motives which could produce them, I would have nearly said, with the habit which one bears. The excellent words of Molière: *Vous êtes orfèvre, monsieur Josse*,¹ is so pleasing only because it is profound, & that it is worth a treatise on this matter. Is it rare to see malady or health, the extent or the proximity of the last term of life to regulate our religious ideas, to give or to take off weight to some sentiments which have certainly nothing to separate from parallel alternatives? Such no longer believe in God when he is carried well, who will begin to believe when his organs weakened or disarranged will expose him to the salutary action of the solitude & of reflection. Throughout where some exterior & independent causes of the investigation act in mass on some entire nations, as climate, education, religion, the form of government, custom & usage, it is established on all the objects near to our knowledge, on all the proceeds of common & civil life a crowd of opinions & of systems, which have for them only a subjective probability & of which one explains the vogue & ascendant only in characterizing the hardness of the soul on which they exercise their empire. The words of Pascal: *Verité & justice en-deça de ce ruisseau; erreur & injustice au de-là*,² has no other origin.

I have indicated finally some individual principles of this consent that obtain from us certain propositions which have in their favor only this disposition alone to admit them. These principles are able to be only some modifications very diversified from these principles, & universals & particulars that we just examined, as the individuality is never but that which, added to the generic & specific determinations, is expressed by no formula & is conceived only by the existence of a given subject. Thus, more or less from these quantities, either, in equal number, the diversity sometimes of degree or of nuance, sometimes of a mixture among them, makes that a man is not another, that he is not similar to himself in all the moments of his existence, & that each of the moments in its duration is different in that from all other. The known observation or rather the general cry: *quot capita tot sensus*!³ that say I, the mobility more or less striking of the sentiments in the same person on the same subjects, according as he envisions them in a different disposition of body & of mind, has its source only in that which there is proper & individual to each thinking being. To the question why such proposition rejected by others, appears probable to such who shortly will embrace the contrary proposition, it has often no other response to make except that it is to him who finds it probable, that he finds it such because it is to him & not another, because being that which it is, occupying in the universe an incommunicable place to all other under all its relations, & seeing the objects in its manner he must also judge in its manner, that which, without doubt, appears to be the triumph of subjective probability.

But as much it matters to well distinguish these two sources of that which we call probability, & to not confound that which comes from the objects with that which comes from us in the probable estimation of the true, so much one will be deceived in holding these two sources much extended from one another, in imagining that the one acts without the other, & that it can consequently have a probability so objective that it had nothing of subjective, or so objective that it had nothing objective. Experience

¹Translator's note: From *L'Amour médecin*, Act I, scene i. *You are goldsmith, Mr. Josse*. The context is that to cure Lucinda, Josse suggests that diamonds be purchased. That is, Josse makes a recommendation in his own self-interest.

²Truth and justice on this side of the stream; error & injustice on the beyond. *Pensées*

³As many heads, so many opinions! Found in *Phormio* 454 of Terence.

& reasoning attest the contrary, & it is in this regard with the spiritual vision of it, if I can express myself thus, as with the corporal vision. The eye without object will not see; the object without the eye destined to see it, will never be perceived; but in the moment where they coexist, vision is operating; it is therefore the result of the qualities of the eye & of the qualities of the object being mixed & acting on one another in order to produce a third object, I just said a phenomenon, which being neither the eye taken in itself nor the object taken in itself, can without doubt give us the idea perfectly just neither of the one nor the other, but does not authorize us at all for that to deny the real & absolute existence either of the two objects at the same time, or of one or of the other. And it is likewise of the relations which subsist between the true & our soul. The truth which, were it only in the governing intelligence, must have an existence proper & independent from our judgments, would be null with respect to us without our soul & the organization which is appropriated to it, & our soul without the truth & supposed that the truth meets with no part, would be without sustenance & without function. You put them in contact; immediately you believe a third object, I wish to say a judgment that I will call here probable in order to contain myself in the matter that I treat. This judgment is only a phenomenon, because without being neither the truth itself nor our soul properly said, & without being able consequently to give us a perfect & intuitive knowledge of both, it is resulted from the mixture & from the reciprocal conflict of the characters which arrive to them, & the ignorance where it leaves us out of their proper nature, separated from the combined game which modifies them, must not prevent us to accord to them a proper & absolute existence. It is this probability where there enters consequently the objective & the subjective, that I will call composite or mixed probability.

These principles whence each probability derives would seem to be able to give the definition. If the objective probability supposes in the subject of which it exposes the predicates, a reunion of these given & absolute predicates, circumscribed by the nature of the subject, independent of all that which we can know of them; & in the usage that we make in order to decompose it, of all the psychological artifices & known logics, a perfection, an infallibility assignable if not for us, at least certainly in itself & to the eyes of governing intelligence, one could define the objective probability the relation that there is on the one side between that which the subject is for us who know it in part, & that which it is in itself; on the other between the degree of justice, of precision & of exactitude which we have put in the application of the rules of reasoning & of the method which are known to us, & the degree of surety of which these rules would be susceptible in themselves & handled by some superior minds. If the subjective probability is contained totally in the impression that a proposition makes on us by some different causes of the power of the truth clearly & distinctly perceived, one could define the subjective probability the relation that there is between the state where the soul is found with any degree of assent & of persuasion, & the state where it is when it believes to have the complete sentiment of the evidence & of certitude. The probability that I have called composed or mixed, keeping the first two, permits not at all a particular definition, but it will serve us to another end in the following of this memoir.

I swear that that which would prick me the most, it would be to be able to join to this definition what I regard rather as moral than as rigorously metaphysical, another

definition based on a reflection that I have made in beginning this discourse; but the difficulty that I find, in removing from me the hope of succeeding, does not remove from me the one of testing it with fruit for myself & and for those who will wish to think after me. I have said that *all that which is not demonstrated, is either possible or probable*; that *the contrary of the probable would remain possible*; that *all that which is probable has been possible, before the probability occurs*; and that *if, under this completely general aspect the probability is only a kind of possibility, it remains to see what is properly the difference that there is between one & the other*. There are thus two questions here; the one: Is it true, & why is it, that probability & possibility would be in a certain sense synonyms? The other: What is it which occurs to the possibility in order to make it a probability? As a probable proposition is that of which the truth or the liaison of the predicate & of the subject is probable, likewise a possible proposition is that of which the truth or the liaison of the predicate & of the subject is possible, & a demonstrated or necessary proposition, that of which the truth or the liaison of the predicate & of the subject is demonstrated or necessary. Everyone agrees that a demonstrated or necessary proposition is that of which the contrary is impossible & implies contradiction, but one would be tempted to believe that a possible proposition is also that of which the contrary is impossible; & however, I must say a paradox, I would believe to be able to advance that a possible proposition is that of which the contrary is possible, & here is on what I base myself. Each enunciation of a demonstrated proposition is simple; the contrary can therefore only be impossible. When I say: This is, or this is not, I present only a single idea; there is no point here to choose neither alternative, nothing which even suggests it, so that in the end there is only the impossible. Each enunciation to the contrary of a possible proposition is double & presents two ideas. When I say: This can be, I suggest by this likewise that this cannot be, & you note that there is no alternative or disjunctive here at all; I have no need to say, & likewise I must not say: This can be, or this cannot be; I say: This can be, & this can not be. It is likewise because one of these assertions is true, that the other is it also; it is because one thing can be, that it can also not be; under the possibility of the first enunciation is contained the possibility of the second. According thus as you will reunite or not reunite in a possible proposition the two ideas that it presents at the same time, that is to say that which it enunciates & that which it suggests, you will be wrong or have reason to define a possible proposition that of which the contrary is impossible. If you take the two parts of the enunciation at the same time, it will be quite true that the contrary of the possible is the impossible; because it is true that a thing can be & can also not be (this which forms the two possibilities) it is quite clear that it is impossible that it may not be & that it may not not be. But if on the contrary you take only one of the two ideas that the proposition presents, it will be very true that the contrary of the possible is again the possible. In saying, & in limiting you to say: This can be; you let the contrary to subsist, that is to say: This can not be, & this contrary is so possible that it is contained in your first enunciation. It is therefore from this second point of view that I depart in order to define a possible proposition, that of which the contrary is possible; & it is also, it seems to me, from that definition that one can depart in order to sustain that in a sense probability is synonymous with possibility. Probability is with respect to certitude or to demonstration, an extreme, an indivisible point, because one can not quit the certain or the demonstrated without also encountering the probable,

but one must represent probability taken in part as an extension, a continuum which is divided as much as one wishes. The probable is opposed to the demonstrated, but this probable contains all the particular probable propositions that one can enunciate. It is among all these propositions that exist that which one calls the degrees of probability, by virtue of which there is such which is not at all of the whole probable & such which begins to be it; & from this here which is the least to that which is the most, exist all those of which the probability goes by increasing or by decreasing, in a manner that between these two series, the one ascendant & the other descendant, there is a unique proposition in its genre & that makes the separation of it by half. If there is therefore no possible proposition at all which permits one of them to subsist which is also so when one enunciates only a part of the alternative; it is equally true that the contrary of a probable proposition is improbable, when one pronounces them both in the same direction, that is to say by going for example for the one & for the other from the *No* supposed impossible, to the *Yes* supposed demonstrated; & that the contrary of a probable proposition is again probable when one pronounces them both in an opposite direction, that is to say by going for the one from the *No* supposed impossible to the *Yes* supposed demonstrated, & for the other from the *Yes* supposed impossible to the *No* supposed demonstrated. I could render this here sensible by a table where one would see always on two scales of probability, any two contrary propositions, to correspond however by the same degree of probability where they are found, because the term to which they end, or the extreme proposition toward which one advances is different; & this is the point of contact where it seems to me that the probability coincides with the simple possibility, so that as one can say that there is a sense where the contrary of the possible is yet the possible, there is one also where one can say that the contrary of the probable is yet the probable. This property is common to the simply possible propositions & to those which are probables, with the differences that we have indicated, & it would be easy to show with respect to the first ones, by tracing in them also two scales according to the two enunciations that the propositions include; but this property does not extend at all to the propositions contradictory among them, because among these last one can imagine only a line continuous & without fractions, instead that the two others sorts of propositions are the only ones which admit a subdivision & a partition of the line which represents them either in different enunciations, if they are only possibles, or in degrees if they are moreover, probables.

However, & despite this rather singular analogy in order to be noted, it is evident that there is a very great difference between a possible proposition & a probable proposition, & that if all the probable propositions are drawn from the vast field of possible propositions, the first ones should undergo a change which has raised them so to speak to this second power. But what is it, this change? How is the passage made from the point where there is no probability yet, to the one where the first degree is manifest of it? Here is the difficulty. There would be nothing to say but to define the probability, the supplement or the complement of the probability, because it is an idea & not a word that we seek; it would be repeating ourselves to set down the definition of it on the elements of probability so much objective as subjective & mixed that we have detailed above; & this would be to alter the notion of the possible than to define the probable that which forms a degree in the possible, because the notion of the possible is one, indivisible & such that in the end of the rigorous idea that it presents, there is nothing

which resembles it; never was it said that a thing was more possible than another unless one had spoken badly; there would remain therefore no more than to define probability a logical & moral fraction of a similarly logical & moral Whole.

However satisfying that this definition may appear to me at this moment because I find no other, I am obliged to swear that it serves almost to nothing in the matter that we treat, & that one is no more advanced in the explication of probability after, than before its discovery. The reason for the non-utility of all that which one calls definition of probability, when even one would call the calculus & geometry to its aid, would it not be the impossibility where we are to determine the ratio which is found between the number & the force of the trials which constitute the probability, & the number & the force of those which are necessary for the demonstration, & to evaluate thus by relation to the whole, the fraction of which any probability gives the idea. More than one consideration explains likewise this impossibility, & can render comprehensible why this portion of the work of the soul on a proposition, & of the result that it seeks & that we call probability, remains always incalculable or incommensurable with the work or the entire result to which it is related & which would constitute certitude.

First, the principle that one poses & that it seems that one is obliged to pose in order to form this numerical evaluation, appears to me abusive. It has a double object; either one wishes to conclude only to the probability of a phenomenon, or one wishes moreover to express the nature & so to speak the amount of this probability.

In the first regard, do you depart from the idea of cause & effect in order to form your probable judgments, either that the return of that which you call cause leads you to sense the effect, or that the nature & the presence of the effect makes you rise to the cause? On one side the Sceptics will prevent you & will demand you reason from the qualification of cause & effect that you give to your phenomena; on the other you gain nothing by this qualification: because if the liaison which you set between cause & effect is necessary, it is no longer a simple probability that determines you to believe; and if it is contingent, cause & effect are synonyms to antecedent & consequent. Therefore all probable propositions to some object that one applies them, be it estimation of human life, gains & losses in games, tontines & bankers, political economy, force & value of testimony, physical & moral phenomena; all the probable consequences repose only on a single principle, it is this one: that which is arrived will arrive again, & will arrive by the reason that it is arrived already; the more a thing is arrived often, the more it is credible that it will arrive again, at least it is it, because the sentiments have been divided on this subject; now this principle or rather this reasoning transformed into principle appears to me vicious & destitute of all force. It presents a fact which is clear, that one can establish & that one establishes every day by some observations, some registers, some tables which one makes very well to prepare with the greatest care, & to continue with the greatest perseverance, it is that such chance has taken place so much & so many times; but that one infers from it that it will take place again or that it will no longer take place, it is without doubt that which it is always very useful to believe (because I do not touch the practical at all) but it is to what makes it, had it been repeated a million times permits it not at all in logical rigor, because some phenomena that one considers simply as following themselves, & placing themselves end to end, are not produced at all, are not brought forth necessarily, no more than make it from the units that you set on one side of the other in such progression as you wish.

To each of them, if you act without intention & without end, it will be possible to you to arrest yourself or to begin a different series. It is necessary therefore, in order to render legitimate the consequence that one draws every day from the reiteration of certain phenomena, to suppose between this repetition & the judgment that it brings forth, any law, either proven or simply adopted, which link them in our mind & supplied to that which the sole return of the object indicates not at all; & I am persuaded that it is that which arrives always without what we ourselves render account of it to ourselves. The subjective probability of which we have exposed the effects, furnishes us these intermediate ideas or these instincts, which being mixed with our observations on the return of the phenomena, may lend them with the force of a principle which they have not at all, of the consequences which are not at all contained there. Man is in general to be entrusting of it; to believe is his natural state; it is pleasant, either by indolence or by sentiment of the feebleness which permits him neither to support the habitual & universal doubt, nor to dissipate it always by demonstration, be it because the basis of constancy, & of uniformity which characterizes & its proper constitution and all that which arrives under his eyes in the physical & moral universe, accustoms him to that security from which he rarely exits, & more by reflection & by system than by taste & by tendency. He believes rather without reason that he will not doubt without reason; it is an undertaking for him to doubt, instead that it is his natural direction to believe: thence comes that he has not always the distinct perception of all the things which he believes, & that it can happen to him to believe much more than he himself not imagine it. You cause a very lively contradiction to him or a persecution on any sentiment; he will defend it with a heat & perhaps a success which will apprehend to it that which he would not know, it is that this sentiment at base was his before the dispute, but that it is lost in the crowd of opinions which without ceasing & on all sort of subjects attach themselves to our soul. He no longer always keeps in mind the reasons which make him believe; many will act on it without knowing it, without being in a state to expose them, & want of noting this circumstance, one will make often the injustice to him to believe that of the principles you grasp very well, but that he has not the talent to justify, are only some prejudices with respect to him. It suffices that a sentiment be evidently neither absurd & contradictory, nor prejudicial at first glance of the eye, in order that he adheres & that he finds singular what one demands from him the proofs of it; if, with that this sentiment is generally received, if it is good to him, necessary even to have it, if it accords with a law of order, of harmony, of unity which pleases him, & as man & as religious man, counts that this sentiment will be profoundly engraved in his soul, without that the idea may be to him perhaps ever come to be examined in all rigor. I apply these reflections to the judgment which declares a proposition probable, & I say that it is not in this succession alone of phenomena contingent whence the world results, that can be found by good logic the foundation of the consequences however very valuable in common life, that we draw from them, but in those tendencies or primitive instincts that I just indicated. Do we demand to ourselves why we believe that the sun will rise tomorrow? It seems that the only reason that makes us believe, is that it is risen already so many times since the origin of the world, & it would be well that we will claim this reason, & that we will appear to satisfy ourselves with it. However as it is worth absolutely nothing, considered closely, I am persuaded that it is the least of those which incline us toward this sentiment, & that these strongest are

those which are drawn from ourselves, from I know not what moral mechanism which leads us to certain conclusions incompatible with the rigor of reasoning. This long sequence of risings of the sun has encountered in our soul a crowd of secret tendencies which it has fortified, & which in their turn have made from them that which it is not at all, a sort of necessary law; it has found us for example, disposed to believe rather than to doubt, to believe especially that which one puts rarely into question, that which it is of use to believe, that which tranquilizes when one believes it, & accords with all our projects & our innate desires; it has found us disposed to believe that that which arrives ordinarily or nearly always without that one see a necessity properly so-called to produce it, holds to some thoughtful views, & this idea of a law of suitability one time admits & deduces from one simple repetition of events, transforms in its turn that repetition into law. To arrive often, nearly always, & to must arrive, confounds itself in our mind. Thus a consequence which is not at all logical, if you seek the reason in the understanding alone & the rigorous analysis of its operations, becomes it anew if you call it, in order to comprehend it, to a man each whole & in fabric so complicated of his tendencies, of his needs & of all his being. It is also true that the millionth rising of the sun concluded nothing, not even probably, in favor of the following when one takes it alone & in itself, that it is true that some millions of units placed the one after the other contains not the reason of that which you can increase the number.

It is necessary to make the same reflection on the second part of that which one calls the principle of probability, I wish to say that which has for object to determine the relation of such sum of degrees of probability to the whole of which one supposes that it makes part. This ratio which without doubt exists *in abstracto* & for a superior intelligence, appears to me imaginary in all our assumptions. Of the lines or of the numbers which are very good in order to design with brevity & clearness that which is, is not able to create that which is not at all & would not know how to be; because finally each proportion demanding two terms of which the known & determined distance renders possible alone the scale which you raise between them, it is clear that by taking off one of these terms, by supposing it unknown & inassignable, the other serves you for nothing & leaves you only an ideal & chimerical calculation; now where is the total & fixed sum of all our givens for any probable conclusion? null part; & by consequence how to decide that the number of the givens that we have, is to that sum as three to four, or four to six, or as one would express it? However long that the sequence of cases or past phenomena be, it serves nothing to establish a proposition, if the *terminus ad quem* is lacking, if I can approach it or defer it at my pleasure, if it does not depend absolutely on me to encounter the fixed & irrevocable place that nature of the things has aligned to it. I am able by a fiction to represent to me this last term as a discrete or continuous given quantity, & by departing from there to express with fractions; but indeed, this quantity not existing, at least with respect to me, all my fractions are lost. If there is a question of the objective probability to which we have given two bases, the nature of the subject, & the application of the logical methods in favor of which we assure ourselves of its attributes, it is clear that as much we know in these two regards where we are with them & the path that we have made, so much we not know the one that it would be necessary to make again, & we can not consequently say in what proportion that which is finds itself with respect to that which could & must be. I know, or I am able to know by experience, how many times since the origin

of the world the sun is risen, the stars have traveled their orbits according to known laws, plants, minerals, animals, all the productions of nature are themselves presented under some forms & with some given quantities; how many times in certain areas or regions the winds have made some ravages, the land has suffered violent commotions or vomited destructive fires; how many times men have lived such number of years, said the truth or ceded to the attraction of delusion; how many times in some combinations called fortuitous, such chances have presented themselves: until here it is necessary only to see, to observe, to count. But in order to go further & to wish to express by numbers or other figures the relation which is found between the number of similar phenomena that I hold & the one of all the phenomena of the same order which can or must probably manifest themselves again, it would be necessary that I held the last link of a chain of which I hold only the anterior part, & of which that which is before me can have only an indeterminate length, that which renders it incapable to decide of the value of that which is known. According to nature, human kind, the theater in a word of the results of which you have taken count, will last more or less a long time (that which a person not know, & that which each can imagine as it pleases him) your conclusions, from infinitely probables, will become less so to you, & at least, more of the all. The three quarters will be no more than the half, & the half only a fourth, if I extend through thought the term which is unknown, & of which I can make that which I wish; as through the inverse operation the fourth will become the half, & the half three fourths, if I bring together by thought the term which by circumscribing the space of it that the concern is to divide, gives alone a sense to these denominations. It is necessary, without exiting again from the objective probability, to say so much of the judgment as one bears on the more or less great regularity of the logical process which have given to us a conclusion. The rules of reasoning that the understanding human has made to himself, all certain as they are, leave unhappily always the probability to subsist on our part, often the greatest likelihood, of being deceived in their application. The compass in each of the directions that it can take is infallible, but the hand which leads it is not so & can be lead astray. By subtracting all the errors & the scorns which can by a thousand different causes slide themselves into our reasonings, one could have the highest degree possible of correctness under the usage of the rules of logic, & the point where infallibility properly so called would exist. This ideal of logical perfection exists somewhere, it is the share of a superior intelligence, but it would be contradictory that we knew it, since the force to understand & to calculate all the possible errors would suppose that to avoid them, that we have not. Now without this distinct knowledge of all that which is necessary in order to form in each given case an infallible conclusion, & without the certitude that one possessed this knowledge, it is yet impossible to determine the degree of probability of any one conclusion. The term of comparison or the term with which your degrees of probability must be found in the ratios of a fourth, of an eighth &c. this term, say I, being, in the case of which I speak an ideal perfection, an infallibility supposed under the application of the rules of logic, & that perfection, that infallibility being undetermined with respect to us, being able consequently to receive & to admit all the possible latitudes, one senses yet that in this mobility of the point of comparison each expression of ratio with it is illusory.

It will not be less, & by the same principle, in the subjective probability. One will be reminded that we have called thus the impression that they make on man in

general, or on such man in particular, some propositions considered independently of the character of truth or of falsity that they bear, & which borrow the power with which they act on us only from certain inherited traits in our nature. There is certainly also a *maximum* in that sort of probability, & we have named it the evidence. But where is this *maximum*? What is the extent of it, the known measure, fixed, invariable? Who can, by traveling all the field of the truths, philosophical, moral, religious, political, to assign for each of them the highest degree of assent & of persuasion that it can & must produce or has produced since it circulates in the world, not only on man in general, but on all the individuals of human kind, according to all the accidental & local differences that I have shown to influence on all the judgments which are not from the cause of the demonstration. The evidence in these sorts of judgments is never but a sentiment or the result of many clear but confused ideas. Each of us in hearing of a probable proposition, & in measure that it is more, experiences a sentiment which warns him that it bears this character; the sentiment never goes without that which indeed does not constitute it, but constitutes its force or its power on us, without a given degree; this degree which is nothing by itself, is distinguished only by comparison. One can therefore agree that imagination & memory in us retracing more or less confusedly some past impressions, & more feebly still the different degrees of vivacity which accompanied them, can put each of us in a state to say what has been among all the assertions which it has been with ability to understand & to appreciate during his life, that of which the probability has most affected it; but from there to the determination & to the distinct expression of the greatest possible force with which one can conceive that a proposition, not susceptible to demonstration, may act either on the human soul in general, or on all the temperings of a soul modified by all imaginable circumstances, there is remotely without doubt, & if remotely as there is rather no commensurable route which leads there; & however, without this general & fixed expression of the highest possible degree of assent to one idea, how to give to the intermediate ideas of the arithmetic or geometric denominations which suppose this higher degree known & also invariably fixed if it is small?

I have shown to the present that the principle from which one departs, either in order to express the probability in general or in order to express the degree with respect to certitude itself, quite far from being susceptible to the evidence which accompanies the arithmetic & geometric enunciations, appears even not to agree with the first rules of reason & a sane logic. I believe to be able to arrive to the same conclusion by an attentive consideration of the object of the probability, or rather of the same elements of that sort of calculus to which it appears to me completely strange. The rather ordinary distinction that one makes between natural signs & artificial signs has always appeared to me frivolous. The same definition which one gives of the natural sign proves that it would be worth more to call cause & effect that which one calls natural signs, since effectively one establishes a natural liaison, that is to say real & necessary, between the natural sign & the thing designated by this sign, that which repels at the first idea which that word awakens in the mind.

But as soon as each sign is arbitrary & supposes a convention, it seems that there is nothing absolutely that one can render by signs. The words of *quality* & of *quantity* which represent the two most universal & most inseparable attributes from all the objects of our understandings appear susceptible only of a reciprocal negative definition,

& I do not know if that which we understand from the quality & from the quantity is extended much to the end from this here; it is that the quality is that which remains in the object when the quantity is removed; as on the contrary the quantity is that which is conceived in the object, setting aside from the quality. This purely negative definition, exclusive of one of the contraries in turn, & which consequently teaches us nothing, is also receivable only in the sphere of abstractions. At the moment where we take the things such as they are & as nature offers them to us, not only the quality & the quantity are found necessarily together, & no object is intelligible without these two attributes reunited, but it is clear further that one can not be defined without the other. If we knew perfectly some quality that this be, we would see that it carries with it its quantity without which one can not conceive it. An idea is never clear or obscure, distinct or confused in general, it is always in such proper & individual degree; a judgment is never true in general, the place which unites the subject & the predicate is always of a certitude or of a given force; remove in a sentiment its energy, its proper heat, this is no more to it than you have, it is another of it; you say Leibniz, Locke, Newton, these are some intellectual quantities which you pronounce. It is likewise of bodies. There is never in the nature of a heavy or light body which is at the same time more or less; not of air, cold, heat, dry or humid which is not all the one of a certain manner, & under this point of view all quality is quantity. In the same sense all quantity will be only a quality which by that likewise that it exists, that it has its individuality proper & pronounced, will be always a quantity exalted to a certain point, gifted of a given degree of force, was it the lowest possible. Weight, cold, heat, spirit, judgment, genius, all these words in their individual application signify nothing without the measure & the degree closely united to the existence of the objects which they designate. As each sign depends on a convention which has no other limits than the needs or the caprice of those who stipulate it, it is evident that the qualities as well as the quantities can be represented by some signs. All the signs which have preceded the language or have accompanied the imperfection of it; all the words of the human language in the diversity of the forms under which there is divided the climates & men, belong to the indication of qualities; & the numbers affected to the discrete quantities, the lines in the continuous quantities, have served to indicate in the beings or modifications of the beings all that which is not quality, all this which, without saying that which a thing is, must say only how many times or until what point & in what measure it is that which one supposes it to be. These signs are susceptible of a double substitution. One substitutes them among them. Is the concern with the qualities of the objects, & of the words or of the different idioms that one regards as some signs of that which are the things? One translates from one language into another & into all the others. Is the concern with the quantities? the numbers are set for the lines & the lines for the numbers; but everyone will agree that this substitution leads to nothing, & that the only veritably useful or rather the only indispensable, if it must extend the sphere of our knowledge, it is the second that I have insinuated, that which consists in realizing the signs & to put finally in their place the same objects that they must make known to us. Throughout where this substitution is impossible, or difficult, or uncertain, the signs serve nearly to nothing. Thus the one who would know in all the ancient & modern languages the name of an animal, or of a plant, or of a process of the arts, & who would never have had occasion to see that animal, that plant, that process, or to make the least idea of it, would know nothing

assuredly; thus pure arithmetic (if I can express myself thus) gives to the mind of the precision, of the justice & of the force, often amuses it & amazes it by the properties hidden in the numbers, & does not go beyond; so much pure geometry crowns these advantages by greater & infinitely greater still, so much it is evident that it is especially of its application to the real objects of thought, of nature, & of the arts that it draws its most solid radiance & its veritable utility.

But it is at the same time this application, according to which it is more or less possible, more or less sure, which establishes between these two sciences the most marked differences. The numbers and the lines either are the discrete & continuous quantities themselves, or are only the signs of them. In the first regard numbers can be applied to the lines & to determine the proportions of them; the lines are counted then to have alone of the value & to correspond to reality, the numbers serve them only with sign & with expression. In the second regard, I wish to say if one gives to the others only the function of signs, the usage that one can make of both is very different. The lines, when one takes them only for the signs of the continuous quantities, or of the extent & of its dimensions, are applied to all bodies with so much facility & of success than of utility. The object to which one applies them is of the same nature as they, it is fixed & known; the senses are judges of it; the view & the touch reunited leave no doubt not only on the general dimensions of each body, but even on those which characterize each body in particular. Nothing here is vague, indeterminate, hidden. If the impression & the idea of the extent are confused, they are at the same time the clearest of all those which we have, the despised here are impossibles. It is not likewise with the numbers. If they are less themselves of the discrete quantities than of the signs & of the representations of discrete quantities, there remains to see that which it is necessary to understand by discrete quantities. Any definition that one gives to it, I presume that the notion of degree must enter, & the degree is nothing other than the quantity of the quality. The difficulty is therefore not to attach any sign to an object or to an existing phenomenon & that this sign does not produce more than the physician or the psychologist, but to be assured by some means independent from this sign, of the existence of the object or of the phenomenon; this phenomenon is here the degree of a quality, because the question is to measure or to weigh the force with which a proposition acts on the mind; now it seems to me that despite the ingenious machines in the favor of which we are arrived not only to discover the properties of the air, of water, of fire, of the earth, but to fix by the calculus & by some signs the quantity in which these properties are found, it must pass for constant that a psychometry or any arithmetic or geometric operation destined to determine the degrees of probability assignable to all the kinds of known propositions, is impossible & contained more in a kind of contradiction. The following considerations that I am only able to indicate, are sensibles. The word degree wakens not even a confused idea, in pressing the terms. One is not represented by a degree, one senses it, & what is that sensing? what is that to sense for the same one who senses? all the more reason for the one who does not experience the same sentiment, & to which the concern will be to give to it by signs a feeble suspicion?

We have in order to make & to multiply some psychological observations neither the occasions, nor the powerful motives & the diverse means, nor the instruments & the facilities which have animated the genius to the research on the principles & on the

most hidden effects that nature conceals in its bosom.

We can arrive to the notion of that which we call degree, or measure, or quantity of a quality, only indirectly & by comparison. If it had been possible that during all the duration of our existence we may have perceived only a single quality & a single degree in this quality, perhaps we would have been able to have an idea of this quality; but by sure coup we ourselves would not have doubted the degree in which it was found, we would not distinguish it from it, & it had not existed at all with respect to us. In the mechanism of our thermometers, barometers &c. there is between the quality of which we wish to know the degree & the conventional measures which we apply to it, an intermediate object on which, by some recognized analogies, this quality acts, & which represents it in the diverse states which it experiences. But where will be with respect to understanding, weighing the truth in all its nuances, or rather modified & nuanced itself by all the aspects that it presents to it, where this milieu will be in which all these variations come to be reflected & to manifest itself in our regard? To what different object of understanding, but at the same time of like nature & more to our capacity, will we apply our graduated scale & the palpable support of the situations which it experiences? The traits of the aspect, the discourse, the rule all the same can deceive & deceive every day, nothing of all that certainly & by supposing a fidelity which is not at all, has a known relation with the corresponding states of the soul, a relation that the concern is only to render sensible to the eyes.

Throughout where the mechanism alone acts one must attend to some fixed laws, constant & invariable, the exceptions must be rare, it must be easy with the help of time & of experience, by the aid of meditation, to make them return in the rule or to give to them some particular explications. But the faculty that a man has & man alone enters among all the productions of nature, to understand the laws of thought which he brings with him by being born, to clarify them or to confuse them, to affirm them or to loosen them, to rule or to pervert, to restrain or to extend the applications of it, to be conformed or to defy them & to violate them, this principle of spontaneity & of liberty, truly incalculable in its effects, if it is never, by a blessing of nature, rather strong in order to reverse all the limits which instill a uniform march to its activity, suffices however in order to put into all its intellectual operations an imprecision that one finds not at all in nature dead or simply gifted with sensibility & of life, & which refuses itself to some fixations & to some calculations by all admissible besides. However composed & complicated as are the qualities of which the physician seeks to determine the quantity & the measure, it is necessary to arrive that nothing resemble to man for the number, the diversity, the action & the continual reaction of all the motives which compete & must compete with him to the formation of the least judgment.

That which we have said above, of the sources or of the principles of probability, in distinguishing it from objective, subjective & mixed probability, shows that it is not at all with the understanding alone of man of whom the march is rather simple & rather known, that engages the one who wishes to subject his probable judgments to some precise rules, but that it is on man complete that bear his researches, on his temperament, his education if different, his local circumstances, his innate tendencies, his passions, his caprices, his tastes, his interests, on all that which he has moreover enemy of order & of regularity. To wish to be carried into this labyrinth by the son of an arithmetic logic, it is to wish to carry into the chaos the compass & the balance

which must in measure & in thought confound the elements.

It is finally never that the general atmosphere that our instruments & our machines must make me understand. You agree that on the side of the impression that we receive from it, it is modified next according to all the diversity of the bodies on which it acts, that each of us under this point of view has its particular atmosphere which makes that the state of the air that you have submitted to some precise estimations remaining the same, one can have heat while the other has cold, one can find weighty or humid air while the other will find light or dry; & it is precisely also against this prodigious diversity & this opposition of minds in fact of probability, that all our calculations will come to fail. The sum of the probabilities that one estimates to be those of all humanity, is composed of the characteristic traits which belong to man of all time & in all places. The sum of the probables for such or such class of men in particular is composed of the reunion of the circumstances of which the continual & uniform action has reassembled them & retained them in the category to which one returns them. The sum of the probables for each individual results from all that which there is proper & unique in his manner to be, to live, to see & to think. These three sorts of elements mix themselves & combine themselves in a thousand different manners in order to give new results, without counting all the modifications that the ones experience through the others, the elements so numerous & so diverse, subordinated to each of these three kinds. You calculate, you think here; where will the certain & universal bases be of your calculation? To what will you have use, since this which is rule for the one, is not & can not be rule for another?

I am nearly ashamed of the consequence to which these reflections may seem must lead me. The idea of a logic of the probables or of a theory properly thus so-called & demonstrable of probability is presented to the most beautiful spirits, & is entered into the sphere of their most interesting works. Some, as Moses Mendelsohn, Garve⁴ & others, have examined to where it was admissible, others, as Leibniz, have outlined the table; others have appeared to realize it mathematically, one knows on this point the work of the celebrated Jacob & Nicolas Bernoulli; others have restored, as to a fabric of demonstrate or demonstrable rules, some the law, others medicine, others the questions which can be born on political arithmetic, the establishment of tontines, of lotteries, of the funds for the widowed, the hazards of the inoculation of small pox, the decisions rendered by the plurality of votes, maritime assurances & all the speculations where demonstration abandons us. Struyck in Holland, Deparcieux in France, Young in England, Euler & Lambert in Germany, Toaldo in Padua,⁵ d'Alembert, Daniel Bernoulli, & our illustrious colleague of the same name in this Academy, would be only one part of the authors that I could name, if I would wish to exhaust the scholarly information that this last has had the complaisance to furnish me on the history of this matter. It seems therefore that one had believed all the time, that one believes yet that of that which observation & experience have taught us on this particular matter to reason from the soul, there could one day be born a veritable science, demonstrated as logic, & sure as geometry; however I have some doubts on the possible execution of this plan, that

⁴Christian Garve (1742–1798) of Breslau, Poland was a well-known philosopher who had a reputation as a popularizer rather than a deep thinker.

⁵Guiseppe Toaldo (1719–1798). Priest, geographer and meteorologist.

I have tried to justify in this memoir, & that I believe to be able to summarize thus in two words with the reasons on which they are founded. The logic of the truth or of the certain is possible, it can form a science, because it has a distinct object or what it can become; this is the reason, the understanding, that one envisions alone & sets aside from all that which embarrasses & obscures its march. The doctrine of the probable does not appear to me to ever be able to form a logic properly so-called, or a science, because it carries on a confused object & that it will remain always; it is man complete with all the generic determinations, specifics, individuals, locales, I would have nearly so-called momentary, which enter into the idea that it is necessary to be made of it, & that at each instant make it reason & to believe other than a severe logic can not permit it. It is not that any wise man is able to despise either the observations destined to divine principles in this doctrine, or the practical consequences which result from them. I have simply examined that which the doctrine of the possibility is not at all, & as much as I can judge it, will never be able to be, I wish to say a science properly so-called; reserving myself to expose it perhaps on another occasion the precious advantages in their relations with those of mathematical & metaphysical certitude.